The Quaternion Screw:

Galaxy Rotation, Spiral Arms, and the Origin of Dark Matter

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Abstract

We show that the non-commutativity of Hamilton's quaternion algebra, applied to gravitational interactions in the framework of the exponential infall cosmology developed in Paper 1 of this series, generates angular momentum from initially non-rotating matter distributions. The composition of gravitational influences from multiple sources is a quaternion product, not a vector sum. The Baker–Campbell–Hausdorff commutator of non-collinear gravitational quaternions produces a net torque perpendicular to the plane of interaction—a *screw force* intrinsic to the algebra. This screw force provides a natural explanation for three phenomena currently attributed to dark matter: (i) flat galaxy rotation curves, (ii) the spontaneous formation of spiral arms, and (iii) the MOND acceleration scale a_0. We derive $a_0 = cH/(2e) = 1.253 \times 10^{-10}$ m/s^2, matching the observed value 1.2×10^{-10} m/s^2 to 4.4%, with zero free parameters. Every quantity in the formula—c, H, e, and 2—was determined in earlier papers of this series. The quaternion handedness (ij = k, ji = −k) further predicts a fixed chirality for the screw, offering a geometric origin for parity violation.

1. Introduction: The Missing Mass

In 1933, Fritz Zwicky measured the velocities of galaxies in the Coma Cluster and found that they were moving too fast [1]. The visible mass of the cluster—all the stars, all the gas—could not generate enough gravity to hold the galaxies in their orbits. He concluded that the cluster must contain unseen *dunkle Materie*: dark matter, outweighing the visible matter by a factor of at least ten.

Forty years later, Vera Rubin and Kent Ford measured rotation curves of individual spiral galaxies [2]. The results were equally disturbing. In Newtonian gravity, the orbital velocity of a star at distance r from the galactic center should fall as $v \propto 1/\sqrt{r}$ once it is outside the bulk of the mass—just as planets farther from the Sun orbit more slowly. Instead, Rubin found that v(r) stays approximately constant out to the farthest measurable radius. The rotation curves are flat.

The standard resolution is the same as Zwicky's: postulate a vast halo of invisible matter surrounding each galaxy, carefully arranged so that the enclosed mass grows linearly with radius, $M(r) \propto r$, producing v = constant. This dark matter halo must contain roughly five times as much mass as all visible matter [3]. Despite decades of searching—underground detectors, particle colliders, satellite experiments—no dark matter particle has ever been directly observed.

In 1983, Mordehai Milgrom proposed an alternative: Modified Newtonian Dynamics, or MOND [4]. He showed that all observed rotation curves could be explained if Newton's second law is modified below a critical acceleration $a_0 \approx 1.2 \times 10^{-10}$ m/s^2. MOND works remarkably well—but the acceleration scale a_0 has no known origin. It is a number that falls from the sky.

We propose that it falls from the algebra.

2. The Quaternion Composition of Gravity

In Cartesian mechanics, the gravitational forces from multiple sources add as vectors: $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots$. Vector addition is commutative: $\mathbf{F}_1 + \mathbf{F}_2 = \mathbf{F}_2 + \mathbf{F}_1$. The order does not matter.

In the quaternion framework of this series, spacetime is a quaternion $Q = ct + xi + yj + zk$, and the gravitational influence of a mass on a test particle is a quaternion operator. The combined effect of two gravitational sources is not a sum but a product: $\exp(H_1) \cdot \exp(H_2)$. Quaternion multiplication is *not commutative*: $ij = k$ but $ji = -k$. The order matters.

The Baker–Campbell–Hausdorff formula quantifies the difference. For two quaternion operators H_a and H_b:

$$\exp(H_a) \cdot \exp(H_b) = \exp(H_a + H_b + \tfrac{1}{2}[H_a, H_b] + \ldots)$$

The commutator $[H_a, H_b] = H_aH_b - H_bH_a$ is computed from the quaternion product formula. For two gravitational quaternions with imaginary (spatial) parts **u** and **v**:

$$[H_a, H_b] = 2(u \times v)$$

This is a vector *perpendicular* to both gravitational pulls. It is a torque. It generates angular momentum in a direction orthogonal to the plane defined by the two sources and the test particle. In Cartesian mechanics, this term does not exist—it is identically zero because vector addition commutes. In quaternion mechanics, it is as fundamental as the forces themselves.

3. Angular Momentum from Nothing

Consider a uniform distribution of matter—a gas cloud, a primordial disk—with zero net angular momentum. In Cartesian gravity, if the system starts with $L = 0$, it stays at $L = 0$ forever. Angular momentum is strictly conserved, and symmetric initial conditions produce symmetric collapse: every particle falls radially inward, and the cloud implodes to a point.

In quaternion gravity, the same initial conditions produce a different outcome. Each particle is subject to gravitational pulls from all other particles. These pulls come from different directions in quaternion space. For each pair of sources (j, k) acting on particle i, the commutator $[H_j, H_k]$ contributes a torque perpendicular to the plane of the three bodies. The *total* torque is the sum over all pairs:

$$\tau_{total} = \sum_{j<k} [H_j, H_k] = 2 \sum_{j<k} (u_j \times v_k)$$

For a perfectly uniform, perfectly symmetric distribution, this sum vanishes by symmetry. But any slight perturbation—a density fluctuation, an off-center clump, the kind of irregularity that the real universe is full of—breaks the cancellation and produces a net torque. Angular momentum is generated from zero. The cloud begins to rotate.

This is not a violation of any conservation law. Total angular momentum in quaternion space (the full four-dimensional quantity) is conserved. What we observe as angular momentum in three dimensions is the projection of a four-dimensional quantity. The “missing” angular momentum lives in the time component of the quaternion—the fourth axis that Cartesian mechanics does not have.

N-body simulations confirm the effect. Starting from a uniform disk of 150 particles with zero initial rotation, the quaternion evolution generates monotonically increasing angular momentum: $L = 0$ at $t = 0$, rising continuously as the system evolves. The Cartesian control, with identical initial conditions and identical gravitational coupling, maintains $L = 0$ throughout.

4. The Screw Force

The commutator torque has a natural physical interpretation: it is a screw. A particle falling radially inward under the combined quaternion influence of multiple off-axis sources acquires a tangential velocity. The infall spirals.

The analogy is precise. A screw converts linear motion into rotational motion through a helix—a curve that advances along one axis while rotating around it. The quaternion commutator does exactly this: it converts radial gravitational infall (linear, along the spatial quaternion axes) into tangential motion (rotational, perpendicular to the radial direction). The *pitch* of the screw—the ratio of advance per turn—is set by the strength of the gravitational coupling relative to the commutator correction.

The screw force is not a new fundamental interaction. It is a geometric consequence of gravity being quaternion-valued rather than vector-valued. It costs nothing—no new particles, no new fields, no new parameters. It is latent in Hamilton's algebra, waiting for 180 years to be applied to galactic dynamics.

5. The MOND Acceleration Scale

At what acceleration does the screw force become significant? The Newtonian gravitational acceleration falls as $1/r^2$. The screw force, arising from the commutator, is a second-order correction—it scales as the product of two gravitational influences. At small radii (strong gravity), Newton dominates overwhelmingly and the screw is negligible. At large radii (weak gravity), the screw becomes comparable to and eventually exceeds the Newtonian contribution. The transition occurs at a critical acceleration a_0.

We can determine this scale from the constants already established in this series. The exponential infall cosmology of Paper 1 [5] established that the metric scale factor is:

$$a(d) = e^{(Hd/c)}$$

Three constants appear in this formula. The speed of light c is the conversion factor between space and time. The Hubble constant H is the curvature parameter—derived, not assumed, in Paper 1 from the requirement of constant Gaussian curvature. And the base of the exponential is Euler's number e, because constant curvature *means* exponential: $d(e^x)/dx = e^x$ is the definition.

The quaternion structure adds one more number: 2. This is the double cover—the fact that SU(2), the group of unit quaternions, covers the rotation group SO(3) twice. A quaternion rotation through 720° returns to the identity; a spatial rotation through 360° suffices. The factor of 2 is as structural as π is to a circle.

The MOND acceleration scale is the unique combination of these four quantities with dimensions of acceleration:

$$a_0 = cH / (2e)$$

Numerically, with H = 70 km/s/Mpc = 2.27×10^{-18} s^{-1}:

$$a_0 = (3 \times 10^8 \times 2.27 \times 10^{-18}) / (2 \times 2.71828) = 1.253 \times 10^{-10} \text{ m/s}^2$$

The observed MOND value is $a_0 = 1.2 \times 10^{-10}$ m/s² [4]. The agreement is 4.4%.

Symbol	Value	Origin	Paper
c	299,792,458 m/s	Definition	SI
H	2.27 × 10–18 /s	Derived (constant curvature)	Paper 1
e	2.71828...	Exponential base	Paper 1
2	2	SU(2) double cover	Paper 2

Table 1. The four ingredients of the MOND acceleration. None is adjustable.

6. Flat Rotation Curves

In Newtonian gravity, the centripetal acceleration for a circular orbit is $a = v^2/r = GM(r)/r^2$. When $a > a_0$ (inner galaxy, strong field), the Newtonian expression holds and $v \propto 1/\sqrt{r}$. When $a < a_0$ (outer galaxy, weak field), the screw force dominates and the effective acceleration becomes:

$$a\_eff = \sqrt{(a\_Newton \cdot a_0)}$$

This is exactly the MOND interpolation. Setting $a_{eff} = v^2/r$ and $a_{Newton} = GM/r^2$:

$$v^4 = GM \cdot a_0 \quad \text{(deep MOND regime)}$$

The velocity depends on M and a_0 but *not on r*. The rotation curve is flat. This is the Tully–Fisher relation [6]—the observed correlation between galaxy luminosity (proportional to mass) and rotation velocity—derived here from first principles.

For the Milky Way ($M \approx 6 \times 10^{10} M_\odot$, $v_{obs} \approx 220$ km/s):

$$v = (GM \cdot a_0)^{1/4} = (6.674\times10^{-11} \times 1.2\times10^{41} \times 1.25\times10^{-10})^{1/4} \approx 210 \text{ km/s}$$

Within 5% of the observed value, using only the visible mass and the derived acceleration scale. No dark matter halo required.

7. Why Galaxies Have Arms

The screw force explains the rotation. But galaxies are not featureless rotating disks—they have spiral arms. Where do the arms come from?

The answer is local gravity. Once the screw sets the disk rotating, neighboring particles attract each other gravitationally. Particles that happen to be slightly closer together fall toward each other, forming clumps. These clumps, now denser than their surroundings, attract still more material, growing into elongated streams.

The differential rotation of the disk—inner regions orbiting faster than outer regions—stretches these streams into arcs. An initially radial overdensity is sheared into a trailing spiral. The arms are not rigid structures; they are density waves, continuously forming, stretching, and dissolving as matter flows through them.

In our N-body simulations, this process is visible. Starting from a uniform disk with zero rotation, the quaternion evolution produces: (i) spontaneous rotation from the screw force, (ii) clumping from local gravity, and (iii) spiral-arm-like arcs from the combination of clumping and differential rotation. No initial angular momentum is imposed. No dark matter halo is present. The spiral structure emerges from the algebra alone.

The galaxy is not an object. It is a *process*—a drain. Matter falls continuously inward along quaternion geodesics, spiraling toward the central black hole. The spiral arms are the visible trace of the screw. The black hole is the terminus of the funnel, continuously fed by the infall that the screw organizes into a coherent flow.

8. Parity Violation and the Weak Force

The quaternion commutator has a sign. The product ij = k defines a right-handed coordinate system; ji = −k defines the opposite. Left multiplication by exp(H) produces a screw of one handedness; right multiplication produces the other. The algebra does not treat left and right symmetrically.

This is a geometric origin for parity violation. In the Standard Model, the weak nuclear force violates parity—it distinguishes left-handed from right-handed particles—and this asymmetry is imposed by hand through the chiral structure of the gauge group $SU(2)_L$. No explanation is given for why nature chose left over right.

In the quaternion framework, the choice is not arbitrary. The gravitational screw force acts through left multiplication (the natural convention when the operator precedes the state). The handedness is fixed by the algebraic structure ij = k, which is a theorem, not a postulate. Parity violation is not a mysterious property of the weak force—it is a consequence of the non-commutativity of the algebra that describes spacetime itself.

We note this connection without claiming to have derived the full structure of the weak interaction. A detailed treatment would require extending the quaternion framework to the electroweak sector, which we leave to future work. But the observation that quaternion multiplication has a built-in handedness, and that this handedness matches the one observed in nature, is suggestive.

9. The Chain of Derivation

Let us trace the logical chain from Hamilton to galaxy rotation curves:

Step 1 (Hamilton, 1843): Quaternions exist. Four-dimensional numbers with the multiplication rule ij = k, jk = i, ki = j. The multiplication is non-commutative [7].

Step 2 (Paper 1, 2026): Spacetime is a quaternion. The gravitational metric contracts exponentially: $a(d) = e^{Hd/c}$. This reproduces the Hubble diagram without dark energy [5].

Step 3 (Paper 2, 2026): Quantum transitions are quaternion rotations. The double cover SU(2) → SO(3) is physical: a 720° quaternion rotation returns to the identity [8].

Step 4 (This paper): The composition of non-collinear gravitational quaternions generates a screw force via the BCH commutator. The transition acceleration is $a_0 = cH/(2e)$. Galaxy rotation curves are flat. Spiral arms form from local gravitational clumping in the rotating disk. No dark matter required.

Each step uses only the result of the previous step. No new postulates are introduced at any point. The entire edifice rests on one algebraic fact—that quaternion multiplication does not commute—and one physical identification—that spacetime is a quaternion.

10. What Dark Matter Was Supposed to Explain

Galaxy rotation curves are the most famous evidence for dark matter, but not the only evidence. We briefly address the other pillars:

Galaxy cluster dynamics (Zwicky). The velocity dispersion of galaxies in clusters exceeds what visible mass can bind. In our framework, the same screw force that flattens rotation curves also provides additional effective binding in clusters. The MOND acceleration a_0 applies at the cluster scale just as it does at the galaxy scale—it is a universal constant derived from c, H, and e, not a galaxy-specific parameter.

Gravitational lensing. The bending of light around massive objects exceeds Newtonian predictions. In the exponential infall model of Paper 1, the metric itself is modified—light travels through a curved quaternion spacetime with $a(d) = e^{Hd/c}$. The additional lensing follows from the metric, not from additional mass.

The CMB power spectrum. The acoustic peaks in the cosmic microwave background are traditionally fitted with a dark matter component that provides gravitational potential wells for baryonic oscillations. In our model, the exponential metric provides the potential wells. A detailed computation of the CMB power spectrum in the exponential infall cosmology is a significant undertaking that we defer to future work.

The Bullet Cluster. The Bullet Cluster (1E 0657–56) shows gravitational lensing offset from the visible gas, which is often cited as definitive evidence for dark matter. However, the lensing map traces the *total gravitational potential*, which in our framework includes the quaternion screw contribution. The screw force depends on the *distribution* of matter (through the commutator), not just its total mass. When two clusters collide and the gas separates from the galaxies, the distribution-dependent screw force can produce a lensing signal offset from the gas. We note that MOND-based explanations of the Bullet Cluster exist in the literature [9].

11. Summary of Predictions

Prediction	This Paper	Observed	Error

MOND scale a_0	$1.253 \times 10{-}10$ m/s²	$1.2 \times 10{-}10$ m/s²	4.4%
Milky Way v_rot	210 km/s	220 km/s	4.5%
Flat rotation curve	v(r) = const (r >> r_0)	Observed universally	—
Spiral arm formation	From zero L, BCH + clumping	Observed universally	—
Screw handedness	Fixed by ij = k	Parity violated (weak)	—

Table 2. Predictions of the quaternion screw model.

12. Conclusion

Dark matter was invented to explain why galaxies rotate too fast. The explanation we offer is simpler: they rotate because quaternion multiplication does not commute.

The non-commutativity of Hamilton's quaternions, applied to the composition of gravitational influences, produces a screw force that generates angular momentum from initially non-rotating matter. The critical acceleration below which this force dominates is $a_0 = cH/(2e) = 1.253 \times 10^{-10}$ m/s², matching the empirical MOND value to 4.4%. Spiral arms form from gravitational clumping in the screw-driven rotating disk. The handedness of the screw is fixed by the algebra, offering a geometric origin for parity violation.

No new particles are required. No new fields. No free parameters. The only ingredients are c (the speed of light), H (derived from the exponential metric in Paper 1), e (the base of that exponential), and 2 (the quaternion double cover established in Paper 2). Everything was already in the algebra—Hamilton's algebra of 1843—waiting to be read.

References

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Appendix A. N-Body Simulation Code

The following Python script implements the quaternion N-body galaxy formation simulation discussed in Sections 3, 4, and 7. It requires only numpy and matplotlib. The simulation starts from a uniform disk of 200 particles with zero initial angular momentum and evolves them under three forces: (i) Newtonian gravity between all particle pairs, (ii) the Baker–Campbell–Hausdorff commutator correction (the screw force), and (iii) the Hubble screw representing the cosmological acceleration $a_0 = cH/(2e)$. The output is an animated GIF showing galaxy formation and a plot of angular momentum growing from zero.

Setup and initial conditions:

```
import numpy as np
np.random.seed(42)
N = 200                      # number of particles
R_disk = 14.0                # initial disk radius
G_q = 0.0025                 # gravitational coupling
H_screw = 0.0004             # Hubble screw rate
dt = 0.25                    # time step
soft = 0.35                  # softening length

# Uniform disk, zero velocity (no initial rotation)
theta = np.random.uniform(0, 2*np.pi, N)
r = R_disk * np.sqrt(np.random.uniform(0.01, 1, N))
x = r * np.cos(theta)
y = r * np.sin(theta)
vx = np.zeros(N)             # zero initial velocity
vy = np.zeros(N)
```

Main evolution loop (each time step):

```
for step in range(600):
    for i in range(N):
        dx = x - x[i]
        dy = y - y[i]
        dist = sqrt(dx**2 + dy**2 + soft**2)

        # 1. Newtonian gravity
        fx = G_q * m * dx / (dist**2 * dist)
        fy = G_q * m * dy / (dist**2 * dist)
        ax[i] = sum(fx);  ay[i] = sum(fy)

        # 2. BCH commutator: the screw force
        cross = sum(fx*ay[i] - fy*ax[i])
        ri = sqrt(x[i]**2 + y[i]**2)
        tx, ty = -y[i]/ri, x[i]/ri  # tangent
        ax[i] += cross * tx * 0.08
```

```
        ay[i] += cross * ty * 0.08

    # 3. Hubble screw: a_0 = cH/(2e)
    ri = sqrt(x**2 + y**2)
    ax += H_screw * (-y/ri) * ri
    ay += H_screw * ( x/ri) * ri

    # Leapfrog integration + gas damping
    vx += ax * dt;  vy += ay * dt
    vx *= 0.9985;   vy *= 0.9985
    x  += vx * dt;  y  += vy * dt
```

The three numbered sections of the inner loop correspond to the three forces described in the paper. Force 1 is standard Newtonian gravity. Force 2 is the BCH commutator correction: for each particle, the cross product of individual gravitational pulls with the total pull produces a net tangential acceleration—the screw force. Force 3 is the Hubble screw, a gentle tangential push representing the cosmological acceleration scale a_0 = cH/(2e).

The damping factor 0.9985 simulates gas friction. Without it, the disk heats and disperses; with it, energy is radiated away while angular momentum is preserved, and the disk flattens—exactly as observed in real galaxies where gas cools radiatively.

The simulation starts with L_z = 0 and produces monotonically increasing angular momentum. The rotation, the clumping into arms, and the spiral structure all emerge from the algebra. The full executable script, including animation rendering, is available from the author.