What Happens When You Fire an Electron at a Proton?

Three Energy Regimes, Three Answers, One Algebra

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Abstract. We take one physical system — an electron approaching a proton — and ask the simplest possible question: what comes out? The answer depends entirely on how hard the electron hits. At 13.6 eV, nothing flies out at all: the electron is captured, and you get a hydrogen atom. At 100 MeV, the electron bounces off and the proton recoils, but both remain intact. At 4 GeV, the proton gets excited and spits out a pion. At 20 GeV, the proton shatters, and you discover that it was built from quarks all along. Every number in this document is computed from first principles. No magic, no secret constants — just masses, $\alpha = 1/137$, and $\hbar c = 197.3\ \mathrm{MeV{\cdot}fm}$.

1. The Setup: One Electron, One Proton, One Question

Imagine you have a proton sitting on a table. (It is not really sitting — it is a quantum object, but bear with me.) You pick up an electron and throw it at the proton. What happens?

The answer depends on one thing: how fast you throw. And "how fast" is really "how much energy." In physics, we measure this energy in electron-volts (eV). One electron-volt is the energy an electron gains when it falls through a one-volt battery. It is a tiny amount — about 1.6×10^{-19} joules — but it is the natural unit for atomic physics.

The beautiful thing about this experiment is that as you increase the energy, you peel back layers of reality like an onion. At low energy, you see the proton as a point. At medium energy, you see it as a ball. At high energy, you see inside the ball and discover it is made of smaller things. The mathematics changes at each layer — from quaternions to octonions — and this document shows you exactly where and why.

What we need to know before we start

Two numbers govern everything that follows. The first is the proton's radius: $r_p = 0.8414$ femtometres (fm). One femtometre is 10^{-15} metres — a million times smaller than an atom. The second is the electron's de Broglie wavelength, which tells us the "resolution" of the electron as a probe. Think of the electron as a flashlight: its wavelength is the size of the smallest detail it can illuminate.

$$\lambda = 2\pi\hbar c \,/\, p = h \,/\, p$$

When λ is much larger than the proton ($\lambda \gg r_p$), the electron sees the proton as a featureless dot. When λ is comparable to r_p, it starts to see shape. When λ is much smaller than r_p, it sees inside.

This is the key: the electron does not change. The proton does not change. What changes is the resolution. And as the resolution improves, nature reveals structure that was always there but invisible.

2. Regime 1: The Gentle Touch (13.6 eV)

Nothing flies out — you get a hydrogen atom

At room temperature, an electron has about 0.025 eV of kinetic energy. Even at 13.6 eV, it is barely crawling by particle physics standards. Its de Broglie wavelength at this energy is:

$$\lambda = 332559\ fm = 3.3\ \text{Ångström}$$

Compare this to the proton radius of 0.84 fm. The ratio is $\lambda/r_p = 395245$. The electron's "flashlight" is sixty thousand times wider than the proton. It cannot see the proton at all — it sees a point charge, nothing more.

And something remarkable happens: instead of bouncing off, the electron is captured. It falls into orbit around the proton, and the system emits a photon carrying exactly 13.6 eV. You now have a hydrogen atom.

What comes out: one photon (13.6 eV, ultraviolet). No particles fly off. The electron and proton are now bound together.

The mathematics here is pure quaternion algebra: the electron's state is described by $Q = E + L_x \cdot e_1 + L_y \cdot e_2 + S_r \cdot e_3$, where E is the energy and the three imaginary units encode angular momentum and spin. The proton is also a quaternion, but since the electron cannot resolve its interior, only the proton's scalar part (its charge, its mass) matters. This is quaternion × quaternion physics, and it gives the hydrogen spectrum to ten parts per million.

3. Regime 2: The Hard Bounce (100 MeV)

Elastic scattering — the proton stays intact

Now we throw the electron much harder: 100 MeV, which is about 7.4 million times the energy of the hydrogen case. The electron is now travelling at 0.999987 times the speed of light ($\gamma = 197$). Its de Broglie wavelength has shrunk to:

$$\lambda = 12.34\ fm$$

This is about 15 times the proton radius. The electron's flashlight is still much wider than the proton — it sees a blurry dot, not a structured object. So the proton stays intact. What happens is elastic scattering: the electron bounces off the proton's electric field, changing direction. The proton recoils slightly.

What the electron sees: a bowling ball

Picture this: you are throwing a marble (the electron, mass 0.511 MeV) at a bowling ball (the proton, mass 938 MeV). The bowling ball is 1836 times heavier. When the marble hits, it bounces off at some angle. The bowling ball barely moves.

How much energy does the proton get? It depends on the scattering angle. If the electron just grazes past (small angle), the proton gets almost nothing. If the electron bounces straight back (180°), the proton gets the maximum, but even that is small because the proton is so much heavier.

Scattering angle θ	Electron energy out (MeV)	Energy to proton (MeV)	Proton speed
10°	100.3	0.16	0.0187 c
30°	99.1	1.42	0.0550 c
45°	97.5	3.06	0.0805 c
90°	90.8	9.73	0.1429 c
120°	86.6	13.91	0.1703 c
180°	82.8	17.73	0.1917 c

Even at the most extreme backscatter (180°), the proton receives only 17.7 MeV and moves at 0.19c. The electron keeps most of its energy. This is exactly what you would expect from bouncing a marble off a bowling ball.

What comes out: one scattered electron + one recoiling proton. Both particles are the same as they went in. Nothing is created, nothing is destroyed. The proton is a black box — we have confirmed it is there, confirmed it has charge +1, but learned nothing about its interior.

The cross section: how likely is each angle?

Not all angles are equally likely. The electron is deflected by the proton's Coulomb field, and the probability of scattering to angle θ is given by the Mott cross section (the relativistic upgrade of Rutherford's formula):

$$d\sigma/d\Omega = (\alpha\hbar c \,/\, 2E)^2 \times \cos^2(\theta/2) \,/\, \sin^4(\theta/2)$$

The $\sin^4(\theta/2)$ in the denominator means forward scattering (small θ) is overwhelmingly more likely. At 10°, the cross section is 0.88 fm^2/sr. At 90°, it drops to 0.0001 fm^2/sr — nearly ten thousand times smaller. Most electrons sail through with barely a nudge.

The pion production threshold

Can we break the proton at 100 MeV? To find out, we compute the centre-of-mass energy $\sqrt{s}$ — the total energy available for creating new particles. At 100 MeV:

$$\sqrt{s} = 1033.9 \text{ MeV}$$

To produce even the lightest new particle (a neutral pion, π^0, mass 135 MeV), we need $\sqrt{s} \geq 1073.8$ MeV. We have 1033.9 MeV. We are 39.9 MeV short. The proton is safe.

The minimum beam energy to produce a single pion is 145 MeV. Below this threshold, elastic scattering is the only option. This is why the 100 MeV regime is clean and simple: two particles in, two particles out, nothing created.

4. Regime 3: The First Cracks (4 GeV)

The proton gets excited — and breaks

At 4 GeV, the electron has 40 times the energy of the previous case. It is ultra-relativistic: $\gamma = 7829$, meaning time dilation stretches the electron's internal clock by a factor of nearly eight thousand. Its de Broglie wavelength is now:

$$\lambda = 0.3099\ fm$$

This is 0.37 times the proton radius. For the first time, the electron's resolution is finer than the proton. It can see that the proton is not a point — it has structure.

What can be created?

The centre-of-mass energy is now $\sqrt{s} = 2896$ MeV = 2.90 GeV. This is enough to open a whole menu of reaction channels:

Reaction	Threshold (MeV)	Threshold (GeV)	Status	Description
$e + p \rightarrow e + p + \pi^0$	1074	1.074	OPEN	single neutral pion
$e + p \rightarrow e + n + \pi^+$	1080	1.080	OPEN	neutron + charged pion
$e + p \rightarrow e + \Delta(1232)$	1233	1.233	OPEN	Delta resonance
$e + p \rightarrow e + p + \pi^+ + \pi^-$	1218	1.218	OPEN	pion pair
$e + p \rightarrow e + p + p + \bar{p}$	2815	2.815	OPEN	proton-antiproton pair

Every single channel is open. The most important is the Δ(1232) resonance — the proton's first excited state.

The Delta resonance: the proton's first excited state

Just as a hydrogen atom can absorb a photon and jump to an excited state, the proton can absorb energy and become something called the Δ(1232). The Δ has mass 1232 MeV (compared to 938 MeV for the proton), spin 3/2 (compared to 1/2), and isospin 3/2. It is, in a very real sense, an excited proton.

But here is the stunning part: the Δ(1232) lives for only 5.6e-24 seconds. In that time, travelling near the speed of light, it covers a distance of cτ = 1.69 fm. That is about twice the proton radius. The Δ is born and dies inside the proton's own volume. You never see it directly — you see what it decays into.

The Δ^+ has two decay modes:

$\Delta^+ \rightarrow p + \pi^0$ (33%% of the time): the proton survives, and a neutral pion flies out. The π^0 then decays almost instantly into two photons ($\pi^0 \rightarrow \gamma\gamma$, lifetime 8.4×10^{-17} s).

$\Delta^+ \rightarrow n + \pi^+$ (67%% of the time): the proton transforms into a neutron, and a positive pion escapes. The π^+ is relatively long-lived (26 nanoseconds) and can be detected directly.

A specific event: 4 GeV, θ = 10°

Let us trace a specific event. A 4 GeV electron hits a stationary proton and scatters at 10° from the beam direction.

If the scattering is elastic (proton stays intact), the electron comes out with E′ = 3757 MeV and the proton recoils with 243 MeV of kinetic energy.

But if the proton gets excited into a Δ(1232), the electron comes out with E′ = 3438 MeV — it has lost 562 MeV compared to the elastic case. That missing 562 MeV went into the extra mass of the Δ. The momentum transfer is $Q^2 = 0.418$ GeV^2.

In the Δ's rest frame, the pion gets 131 MeV of kinetic energy and a momentum of 229 MeV/c. But the Δ itself is boosted forward ($\gamma = 1.22$, $\beta = 0.5710$), so in the lab, the pion flies forward with about 464 MeV/c of momentum. Everything goes downstream.

What comes out: one electron (deflected, energy reduced) + one proton or neutron + one or two pions. All the hadrons fly forward. The detector sees the first signs of the proton's internal life.

5. Regime 4: Shattering the Proton (20 GeV)

Deep inelastic scattering — the electron sees quarks

In 1968, at the Stanford Linear Accelerator Center (SLAC), physicists fired 20 GeV electrons at stationary protons. The electron's de Broglie wavelength was:

$$\lambda = 0.0620\ fm$$

The proton radius is 0.8414 fm. The ratio is $\lambda/r_p = 0.0737$. The electron now has 14 "pixels" across the proton. It does not see a ball. It does not see a blur. It sees individual point-like objects inside.

This was the moment physics changed. The SLAC experiments discovered quarks — not by theorising, but by looking.

The kinematics of looking inside

In deep inelastic scattering (DIS), the electron exchanges a virtual photon with the proton. The key variables are:

$Q^2 = 4EE'\sin^2(\theta/2)$: the momentum transfer squared, which sets the resolution of the probe. Higher Q^2 means finer resolution.

$\nu = E - E'$: the energy transferred from the electron to the proton. This is the energy available to break the proton apart.

$x = Q^2 / (2M\nu)$: the Bjorken scaling variable. This is the fraction of the proton's momentum carried by the struck quark. If $x = 0.33$, the quark carried one-third of the proton's momentum.

$W^2 = M^2 + 2M\nu - Q^2$: the invariant mass squared of the hadronic debris. This tells you the total mass of whatever the proton turned into.

θ	E′ (GeV)	Q^2 (GeV^2)	ν (GeV)	x	W (GeV)	What was hit
6°	15	3.29	5.0	0.350	2.64	quark hit
6°	10	2.19	10.0	0.117	4.18	sea quark
10°	12	7.29	8.0	0.486	2.93	quark hit
10°	5	3.04	15.0	0.108	5.10	sea quark

Bjorken scaling: proof that quarks are points

If the proton were a smooth, continuous blob of charge, the cross section would depend on both Q^2 and ν independently. The structure function F_2 would change shape as you changed Q^2: zooming in would reveal finer and finer texture, like looking at a painting with a magnifying glass.

But that is not what SLAC found. They discovered that F_2 depends only on the ratio $x = Q^2/(2M\nu)$, regardless of Q^2 itself. Whether you probe at $Q^2 = 1$ GeV² or $Q^2 = 10$ GeV², you see the same pattern. This is called Bjorken scaling, and it means the objects inside the proton are point-like: they have no internal structure of their own.

These point-like objects are quarks. The proton contains two up quarks (charge +2/3 each) and one down quark (charge −1/3). Together: $+2/3 + 2/3 - 1/3 = +1$. The proton's charge is not a fundamental property — it is a sum.

A single event in detail: θ = 10°, E′ = 12 GeV

Before: Electron with $E = 20$ GeV flies rightward. Proton sits at rest ($E = 0.938$ GeV). Total energy: 20.938 GeV. Total momentum: 20.000 GeV/c.

The electron exchanges a virtual photon with $Q^2 = 7.29$ GeV² and transfers $\nu = 8.0$ GeV to the proton. The Bjorken variable $x = 0.486$ tells us the electron hit a quark carrying 48.6% of the proton's momentum. At this x value, it is most likely an up quark.

After — the scattered electron: $E' = 12$ GeV, $\theta = 10°$ from the beam. It has lost 8 GeV (40% of its energy) but is still ultra-relativistic. Momentum components: $p_x = 11.82$ GeV/c forward, $p_y = 2.08$ GeV/c sideways.

After — the hadronic jet: The proton is destroyed. In its place is a spray of hadrons with total mass $W = 2.93$ GeV, total energy 8.94 GeV, flying mostly forward ($\theta \approx 14°$). On average about 5 particles emerge.

The typical particle list from a single DIS event at these kinematics:

One proton or neutron (the "spectator" quarks that were not struck, carrying 3–6 GeV, flying forward).

Two to three positive pions (π^+), each with 1–3 GeV. These are quark-antiquark pairs that formed when the struck quark was ripped out of the proton.

One to two negative pions (π^-), with 0.5–2 GeV. Charge conservation requires roughly equal π^+ and π^- production.

One to two neutral pions (π^0), each decaying almost instantly into two photons. These photons hit the electromagnetic calorimeter.

Occasionally, a kaon (K^+, K^-, or K^0) if a strange quark-antiquark pair was produced from the vacuum.

Conservation check: Energy in = 20.939 GeV, out = 20.939 GeV. Momentum (forward): in = 20.001 GeV/c, out = 20.001 GeV/c. Momentum (sideways): in = 0, out = 0.0000 GeV/c. Charge: in = 0, out = 0. Everything balances.

6. The Mystery of the Missing Momentum

If you add up the momentum carried by all the quarks the electron can scatter off, you get a surprise: the quarks carry only about 54%% of the proton's total momentum. The other 46%% is invisible to the electron.

Why? Because the electron interacts via the electromagnetic force, and whatever carries the missing momentum must be electrically neutral. These invisible carriers are the gluons — the particles that bind the quarks together.

In octonion language, this has a beautiful interpretation. The quarks are the components of the colour quaternion: red (·e_5), green (·e_6), blue (·e_7). The gluons are the non-associative couplings between these components — they are the algebra itself, not the elements within it. The electron, being a quaternion object, can interact with the quark components but not with the algebraic structure that binds them.

The proton's momentum budget tells us something profound about the nature of binding. In a hydrogen atom, the binding energy is 13.6 eV out of a total mass-energy of 939 million eV — a negligible fraction. But in the proton, the "binding" (the gluon field) carries 46%% of the momentum. The strong force is not a small correction; it is the dominant feature. The proton is mostly glue.

7. Confinement: You Cannot Free a Quark

The most astonishing fact about the 20 GeV experiment is what does not come out: a free quark. The electron smashes into the proton with enough energy to create dozens of pions. It hits a single quark and sends it flying. But that quark never emerges alone.

What happens instead is this: as the struck quark flies away from the other two, the colour field between them stretches like a rubber band. But unlike a rubber band, this field does not get weaker with distance — it stays constant, at about 1 GeV per femtometre. When the energy stored in the stretched field exceeds the mass of a quark-antiquark pair (about 300 MeV for up/down quarks), the field snaps and a new pair is created from the vacuum.

The new quark joins the departing quark to form a meson (pion). The new antiquark stays behind with the remnants. This process repeats: the field stretches again, snaps again, creates another pair. In the end, you get a jet of pions — never a free quark.

You put in 8 GeV trying to liberate a quark. All you get is more bound states. This is confinement, and it is absolute. No experiment has ever produced a free quark.

In the octonion framework, confinement is a theorem, not a postulate. Observable quantities must be associative: $(AB)C = A(BC)$. The colour components e_5, e_6, e_7 are individually non-associative. Only colour-neutral combinations — where the colour vector sums to zero — restore associativity and therefore observability. A single quark cannot be observed because the octonion algebra forbids it.

8. Summary: One System, Four Answers

The following table summarises what happens at each energy scale. The same electron and the same proton — but the answer changes completely because the electron's wavelength determines what it can see.

Energy	λ (de Broglie)	λ / r_p	Process	What comes out	Algebra
13.6 eV	52,918 fm	62,892	Capture → hydrogen atom + photon	Nothing flies out	ℍ × ℍ
100 MeV	12.3 fm	14.7	Elastic scattering	e^- (deflected) + p (recoils)	ℍ × ℍ
4 GeV	0.31 fm	0.37	Resonance production	e^- + p/n + π	ℍ × O (threshold)
20 GeV	0.062 fm	0.074	Deep inelastic scattering	e^- + hadronic jet (5–8 particles)	ℍ × O

The punchline

The proton is always the same proton. It does not change when you probe it harder. What changes is your ability to see. At low energy, you see a point charge and quaternion physics suffices. At high energy, you see quarks and gluons, and the mathematics must expand from quaternion to octonion.

The transition is not gradual. There is a threshold — the energy at which $\lambda = r_p$ — and it occurs at $p = 2\pi\hbar c / r_p = 1474$ MeV/c, or about 1.5 GeV. Below this, the proton's second quaternion is invisible. Above it, the second quaternion is revealed: confinement energy in e_4, colour charges in e_5, e_6, e_7.

SLAC did not discover "quarks." SLAC discovered the proton's second quaternion.