The Strong Force as Geometric Necessity

Octonions, Colour Confinement, and the Algebra of the Nucleus

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This is the fourth paper in the series "It Is All One." In [1] we showed that cosmological redshift follows from exponential metric contraction. In [2] we showed that the quantum leap is a quaternion rotation, and E = hf is a geometric identity. In [3] we showed that the Pauli exclusion principle is a theorem of quaternion algebra, and reproduced sixty spectral lines of hydrogen and helium from first principles. Here we climb one rung higher on the algebraic ladder—from quaternions to octonions—and show that the strong force, colour confinement, and the internal structure of the proton are geometric consequences of the same algebra that gave us quantum mechanics.

Keywords: octonions, quaternions, Cayley-Dickson construction, strong force, colour confinement, SU(3), gluons, proton structure, division algebras, unified field theory

1. Introduction

"The existing formalism is exactly right, but its interpretation is upside down."

— This paper

In the first three papers of this series, we made a single bet: that the mathematics of physics is not a convenient description of nature but nature itself. Complex numbers are not a trick for solving differential equations—they are the algebra of the plane. Quaternions are not a compact notation for rotations—they are the algebra of three-dimensional space, and their rotation rate is literally the photon frequency. The Pauli exclusion principle is not an empirical postulate—it is a theorem that follows from the anticommutativity of quaternion units in three lines of algebra.

That bet paid off: sixty spectral lines, two atoms, all from one identity. But it also left a conspicuous hole. Quaternions have four components—one real and three imaginary. They account for the electron's energy and its three angular-momentum degrees of freedom. They do not account for what happens inside the nucleus. The proton is not a point. It has internal structure: three quarks, eight gluons, a colour charge that is nothing like electric charge. The quaternion framework that works so beautifully for the electron is silent about the quarks.

This silence is not a failure. It is a signpost. Quaternions are four-dimensional. The next division algebra is eight-dimensional. It is called the octonions, and it was discovered by John Graves in 1843—the same year Hamilton carved the quaternion equation into Brougham Bridge—and independently by Arthur Cayley in 1845. The octonions are strange: they are non-commutative (like quaternions) and also non-associative. That is, $(ab)c \neq a(bc)$ in general. For a century and a half, they were a mathematical curiosity with no known application in physics.

We will show that the octonions are not a curiosity. They are the algebra of the strong force. The quaternion, which gave us the electron, is a special case of the octonion—it is an octonion with four of its eight components set to zero. When those four components are nonzero, the algebra produces exactly the symmetry group of the strong force: SU(3). Eight gluons correspond to the eight generators of SU(3). Three colour charges correspond to three imaginary units of a second quaternion nested inside the octonion. And colour confinement—the mysterious fact that quarks can never be observed in isolation—follows from the non-associativity of the octonion algebra as a geometric theorem, not a postulate.

If this is correct, then the strong force is not a separate force that nature bolted onto electromagnetism as an afterthought. It is the same algebra, one rung higher. It is all one.

2. The Cayley–Dickson Ladder: From Counting to the Nucleus

Mathematics has exactly four division algebras—systems of numbers where you can add, subtract, multiply, and divide without ever getting zero from nonzero factors. They are:

Algebra	Symbol	Dimension	Basis Elements	What You Lose
Real numbers	$\mathbb{R}$	1	1	Nothing
Complex numbers	$\mathbb{C}$	2	1, i	Ordering (no < or >)
Quaternions	$\mathbb{H}$	4	1, i, j, k	Commutativity ($ab \neq ba$)
Octonions	$\mathbb{O}$	8	$1, e_1, e_2, \ldots, e_7$	Associativity ($(ab)c \neq a(bc)$)

This is not a menu where we choose what we like. The Hurwitz theorem (1898) proves these are the only four. There is no five-dimensional or six-dimensional division algebra. There is no sixteen-dimensional one either—the sedenions exist but they have zero divisors, which means they are not a division algebra. Nature has exactly four rungs on this ladder, and then it stops.

Each rung is built from the one below by the Cayley–Dickson construction. The recipe is simple: take two copies of the algebra you have, and define a new multiplication rule that couples them. Concretely:

2.1 From Real to Complex

Take two real numbers (a, b) and define:

$$(a, b) \times (c, d) = (ac - db, da + bc) \qquad (1)$$

This is just the complex number a + bi. Multiplication is commutative and associative. You lose the ability to say which number is "bigger"—there is no ordering on the complex plane—but you gain something extraordinary: every polynomial equation has a solution (the fundamental theorem of algebra). The plane is algebraically complete.

2.2 From Complex to Quaternion

Take two complex numbers (a, b) and define:

$$(a, b) \times (c, d) = (ac - d^*b, da + bc^*) \qquad (2)$$

where * denotes complex conjugation. This gives the quaternion a + bi + cj + dk. You lose commutativity: ij = k but ji = −k. In exchange, you gain the ability to represent rotations in three-dimensional space. This is the algebra we used in [2] and [3] to derive the quantum leap and the Pauli exclusion principle.

2.3 From Quaternion to Octonion

Take two quaternions (Q_1, Q_2) and define:

$$(Q_1, Q_2) \times (Q_3, Q_4) = (Q_1Q_3 - Q_4^*Q_2,\ Q_4Q_1 + Q_2Q_3^*) \qquad (3)$$

This gives the octonion, an eight-component object:

$$O = a_0 + a_1e_1 + a_2e_2 + a_3e_3 + a_4e_4 + a_5e_5 + a_6e_6 + a_7e_7 \qquad (4)$$

But notice what the Cayley–Dickson construction actually does: it takes two quaternions and welds them together. The first quaternion $Q_1 = a_0 + a_1e_1 + a_2e_2 + a_3e_3$ lives in the subspace $\{1, e_1, e_2, e_3\}$. The second quaternion $Q_2 = a_4 + a_5e_5' + a_6e_6' + a_7e_7'$ lives in the subspace $\{e_4, e_5, e_6, e_7\}$. The octonion is not "a quaternion plus four extras." It is two quaternions, nested.

You lose associativity: $(e_1e_2)e_4 \neq e_1(e_2e_4)$ in general. This sounds catastrophic. How can you do physics with a multiplication that depends on the order you group things? The answer is: you do the physics of the strong force. As we will show, the non-associativity is not a bug—it is the mechanism of colour confinement.

2.4 And Then It Stops

You can apply the Cayley–Dickson construction one more time and get the sixteen-dimensional sedenions. But they have zero divisors—two nonzero sedenions whose product is zero—so they are not a division algebra. Hurwitz proved this is not a limitation of the construction; it is a theorem about mathematics itself. There are exactly four division algebras, period.

Here is the physical claim: each division algebra corresponds to a regime of physics.

Algebra	Dim	Physics	What It Describes
ℝ **(Reals)**	1	Classical mechanics	Scalar quantities: mass, temperature, time
ℂ **(Complex)**	2	Waves / circuits	Phase and amplitude: AC current, wave functions
ℍ **(Quaternions)**	4	Quantum mechanics	Q_1 = (Energy, L_x, L_y, S_z)
□ **(Octonions)**	8	Strong force	$Q_1 \oplus Q_2$ = (E, L_x, L_y, S_z) ⊕ (E_conf, r, g, b)

The first three rows are not controversial—they are standard physics, repackaged. The fourth row is the claim of this paper. Note the structure: the octonion row contains two quaternions side by side. The first is the familiar spatial quaternion. The second is a colour quaternion with its own scalar and its own three-vector.

3. Quaternions Inside Octonions: The Embedding Theorem

Before we do anything with the strong force, we must prove a purely mathematical fact: every quaternion is an octonion with four components set to zero. If our quaternion calculations in [2] and [3] are correct—and they are, to 10 parts per million over sixty spectral lines—then the octonion representation of those same calculations must reduce to the quaternion result identically. No approximation. No correction. Identically.

This is easy to verify. An octonion has eight components:

$$O = a_0 + a_1e_1 + a_2e_2 + a_3e_3 + a_4e_4 + a_5e_5 + a_6e_6 + a_7e_7$$

A quaternion has four:

$$Q = w + xi + yj + zk$$

The embedding is:

$$i \rightarrow e_1, \quad j \rightarrow e_2, \quad k \rightarrow e_3, \quad \text{and} \quad a_4 = a_5 = a_6 = a_7 = 0 \qquad (5)$$

Under this embedding, the quaternion multiplication table is exactly preserved:

$$e_1e_2 = e_3 \quad (\text{just as} \quad ij = k)$$

$$e_2e_1 = -e_3 \quad (\text{just as} \quad ji = -k)$$

$$e_1^2 = e_2^2 = e_3^2 = -1 \quad (\text{just as} \quad i^2 = j^2 = k^2 = -1)$$

So the octonion algebra, restricted to the subspace $\{1, e_1, e_2, e_3\}$, is exactly the quaternion algebra. The electron's quaternion is an octonion with its second quaternion set to zero. The hydrogen spectrum we computed in [2] is an octonion calculation with the colour quaternion dormant.

This is the key insight. The electron does not "know" it is secretly an octonion—it has no colour charge, no confinement energy. Its second quaternion is zero, and zero times anything is zero. The strong force is invisible to it. But the quark—which does have colour charge—needs the second quaternion. The quark is the full octonion.

Think of it this way. A shadow on the wall is a two-dimensional projection of a three-dimensional object. The shadow is perfectly real—it has a definite shape, you can measure it. But it contains less information than the object that casts it. The first quaternion is the shadow. The full octonion is the object. For the electron, the shadow is enough because its colour quaternion is zero. For the quark, you need both.

4. Two Nested Quaternions: The Architecture of the Octonion

"The octonion is not a quaternion with four extras bolted on. It is two quaternions, face to face, coupled by a multiplication rule that makes the whole greater than the sum of its parts."

— This paper

The Cayley–Dickson construction builds the octonion from two quaternions. This is not a notational convenience—it is the architecture. Let us spell it out.

4.1 The Spatial Quaternion: What the Electron Uses

The first quaternion lives in the subspace $\{1, e_1, e_2, e_3\}$. We have already identified its physical meaning in [2]:

$$Q\_spatial = E + L\_x \cdot e_1 + L\_y \cdot e_2 + S\_z \cdot e_3 \qquad (6a)$$

The real part (1-direction) is the energy—a scalar, invariant under spatial rotations. The three imaginary parts (e_1, e_2, e_3) are the angular momenta—a vector that rotates. The relationship between scalar and vector is the relationship between energy and spin. The real part is what you measure when you stop the rotation. The imaginary parts are the rotation itself.

For the electron, this is the whole story. Q_spatial is the complete description. The second quaternion is zero.

4.2 The Colour Quaternion: What the Quark Adds

The second quaternion lives in the subspace $\{e_4, e_5, e_6, e_7\}$. Now here is the structural insight that the Cayley–Dickson construction hands us for free: this second quaternion has exactly the same architecture as the first. It has a scalar part and a three-vector part. The unit e_4 is to (e_5, e_6, e_7) what 1 is to (e_1, e_2, e_3).

$$Q\_colour = E\_conf \cdot e_4 + r \cdot e_5 + g \cdot e_6 + b \cdot e_7 \qquad (6b)$$

The "real part" of the colour quaternion—the e_4 component—is the colour scalar: the quantity that is invariant under colour rotations, just as energy is invariant under spatial rotations. What is this colour scalar physically?

It is the confinement energy.

The proton's mass is 938.272 MeV/c². The three quark masses add up to about 9 MeV—less than 1% of the total. The remaining 99% comes from the energy of the colour field itself: the kinetic energy of quarks bouncing inside the proton, the gluon field energy, the quantum fluctuations of the colour vacuum. This is the confinement energy E_conf, and it lives in the e_4 component—the scalar of the colour quaternion.

The three imaginary parts—the e_5, e_6, e_7 components—are the three colour charges: red, green, blue. They form a three-vector in colour space, just as (L_x, L_y, S_z) form a three-vector in physical space. They are the rotation. The colour scalar E_conf is what remains when the colour rotation stops—the energy locked inside the proton by the act of confinement.

4.3 The Full Octonion: Two Quaternions Coupled

The complete octonion for a quark is the sum of both quaternions:

$$Q\_quark = Q\_spatial + Q\_colour \qquad (6)$$

$$= E + L\_x \cdot e_1 + L\_y \cdot e_2 + S\_z \cdot e_3 + E\_conf \cdot e_4 + r \cdot e_5 + g \cdot e_6 + b \cdot e_7$$

For an electron: E_conf = r = g = b = 0. The colour quaternion vanishes. The octonion reduces to the spatial quaternion of [2]. For a quark: E_conf ≠ 0, and exactly one of (r, g, b) is nonzero. The full octonion is active.

Now look at the symmetry. Each quaternion has:

	Spatial Quaternion	Colour Quaternion
Scalar (invariant)	E (energy)	E_conf (confinement energy)
Vector component 1	L_x (angular momentum)	r (red charge)
Vector component 2	L_y (angular momentum)	g (green charge)
Vector component 3	S_z (spin)	b (blue charge)

What the scalar is	What you measure at rest	What makes the proton heavy
What the vector is	How the electron spins	How the quark is coloured

The architecture is identical. The scalar is the invariant—the thing that doesn't change under rotations. The three-vector is the thing that rotates. In space, the rotation is physical rotation (spin, orbital motion). In colour space, the rotation is colour rotation (the SU(3) gauge transformations of the standard model). The Cayley–Dickson construction does not invent this symmetry—it enforces it. The second quaternion must have a scalar and a three-vector, because that is what a quaternion is.

This is why the strong force has three colour charges, not two or four. Three is the number of imaginary units in a quaternion. The colour quaternion inherits the structure of the spatial quaternion, including its dimensionality. Three colours, three spatial directions. The same number, for the same algebraic reason.

5. Eight Gluons, Eight Units: The Colour Algebra

The standard model describes the strong force using the symmetry group SU(3)—the group of 3×3 unitary matrices with determinant 1. This group has eight generators, called the Gell-Mann matrices λ_1 through λ_8. Each generator corresponds to one gluon. This is not a coincidence we are about to explain; it is a fact of the standard model that we are about to derive from octonion algebra.

An octonion has seven imaginary units: e_1 through e_7. But we said there are eight gluons. Where does the eighth come from? The answer lies in the automorphism group of the octonions.

5.1 The Octonion Multiplication Table: Seven Cyclic Triples

The multiplication of the seven imaginary octonion units is defined by seven cyclic triples—index cycles that specify which three basis elements multiply to give the identity, and in which order. These seven triples are the octonion algebra itself; they are not an external diagram imposed on the algebra, but the multiplication table written in its most compact form.

The seven cyclic triples of the octonion multiplication table are:

Line	Units	Rule	Quaternion?
1	e_1, e_2, e_3	$e_1e_2 = e_3$ (and cyclic)	Q_spatial
2	e_1, e_4, e_5	$e_1e_4 = e_5$ (and cyclic)	Cross-coupling
3	e_2, e_4, e_6	$e_2e_4 = e_6$ (and cyclic)	Cross-coupling
4	e_3, e_4, e_7	$e_3e_4 = e_7$ (and cyclic)	Cross-coupling
5	e_1, e_7, e_6	$e_1e_7 = e_6$ (and cyclic)	Cross-coupling

6	e_2, e_5, e_7	$e_2e_5 = e_7$ (and cyclic)	Cross-coupling
7	e_3, e_6, e_5	$e_3e_6 = e_5$ (and cyclic)	Cross-coupling

Notice: Line 1 is the spatial quaternion subalgebra $\{e_1, e_2, e_3\}$—the same multiplication table as {i, j, k} from Hamilton. The other six lines all couple spatial units (e_1, e_2, e_3) to colour units (e_4, e_5, e_6, e_7). They are the cross-couplings between the two quaternions. It is these cross-couplings that create the non-associativity, and it is the non-associativity that creates confinement.

5.2 From Seven Units to Eight Gluons

The automorphism group of the octonions—the group of transformations that preserve the multiplication table—is the exceptional Lie group G_2, which has dimension 14. Inside G_2 sits SU(3), the symmetry group of the strong force, as a subgroup.

How does SU(3) emerge? Fix the spatial quaternion subalgebra $\{1, e_1, e_2, e_3\}$—the electron's space—and ask: which automorphisms of the octonions leave this subspace invariant? The answer is precisely SU(3), acting on the colour quaternion subspace $\{e_4, e_5, e_6, e_7\}$. This is not a metaphor. It is a theorem, proved by Élie Cartan in 1914.

$$Aut(\square) = G_2 \quad \supset \quad Stab(\{1, e_1, e_2, e_3\}) = SU(3) \qquad (7)$$

SU(3) has eight generators. In the standard model, each generator corresponds to a gluon. In the octonion framework, these eight generators arise from the seven imaginary units plus the structure constants of the multiplication table. The eighth degree of freedom comes from the diagonal generator—the one that distinguishes red from green from blue without changing any of them. In the Gell-Mann basis, this is λ_8, the diagonal matrix proportional to diag(1, 1, −2).

Let us count carefully:

- Three colour-changing gluon pairs: e_5, e_6, e_7 (and their conjugates, giving 6)
- Two diagonal gluons: from the e_4 scalar structure and the Cartan subalgebra (giving 2)
- **Total: 8 gluon states = 8 generators of SU(3)**

5.3 Do It Yourself: The Octonion Multiplication Table

You can verify every claim in this section with nothing more than the multiplication table below. Each entry gives the product e_a × e_b. Read the row label as the left factor and the column label as the right factor.

Table 1: The Octonion Multiplication Table (e_a × e_b)

×	e_1	e_2	e_3	e_4	e_5	e_6	e_7

e_1	-1	$+e_3$	$-e_2$	$+e_5$	$-e_4$	$-e_7$	$+e_6$
e_2	$-e_3$	-1	$+e_1$	$+e_6$	$+e_7$	$-e_4$	$-e_5$
e_3	$+e_2$	$-e_1$	-1	$+e_7$	$-e_6$	$+e_5$	$-e_4$
e_4	$-e_5$	$-e_6$	$-e_7$	-1	$+e_1$	$+e_2$	$+e_3$
e_5	$+e_4$	$-e_7$	$+e_6$	$-e_1$	-1	$-e_3$	$+e_2$
e_6	$+e_7$	$+e_4$	$-e_5$	$-e_2$	$+e_3$	-1	$-e_1$
e_7	$-e_6$	$+e_5$	$+e_4$	$-e_3$	$-e_2$	$+e_1$	-1

Check the upper-left 3×3 block (spatial quaternion): $e_1e_2 = +e_3$, $e_2e_1 = -e_3$, and all diagonal entries are -1. This is exactly Hamilton's multiplication table. The spatial quaternion sits inside the octonion, undisturbed.

Now check the non-associativity. Pick three units that span both quaternions, say e_1, e_2, e_4:

$$(e_1 \times e_2) \times e_4 = e_3 \times e_4 = e_7 \qquad \text{(8a)}$$

$$e_1 \times (e_2 \times e_4) = e_1 \times e_6 = -e_7 \qquad \text{(8b)}$$

The two results differ by a sign! This is the non-associativity at work. It arises only when a product mixes units from both quaternions—when space talks to colour. Within each quaternion alone, associativity holds. The non-associativity is the coupling between the two quaternions, and it is the geometric origin of colour confinement.

6. Confinement as Geometric Necessity

"No one has ever seen a free quark. We claim this is not an empirical accident but a mathematical theorem."

— This paper

The deepest mystery of the strong force is confinement. Quarks are never observed alone. Pull two quarks apart and the energy stored in the gluon field between them grows linearly with distance—until there is enough energy to create a new quark–antiquark pair from the vacuum. You end up with two hadrons, not one free quark. Every experiment ever performed confirms this. But the standard model does not derive it—it is put in as an assumption, supported by lattice QCD numerical simulations.

In the octonion framework, confinement is a theorem. Here is the argument.

6.1 Observability Requires Associativity

What does it mean to observe a particle? It means to measure its properties—energy, momentum, spin—and get definite, reproducible numbers. In the algebraic framework, a measurement is a product of the particle's state with a detector's state. For the measurement to be unambiguous, it must not matter how we group the factors:

$$(\textit{Detector} \times \textit{Particle}) \times \textit{Environment} = \textit{Detector} \times (\textit{Particle} \times \textit{Environment}) \qquad (9)$$

This is associativity. If the algebra is non-associative, the measurement result depends on how we group the factors—which means the measurement is ambiguous. The particle cannot be isolated and measured independently of its context.

For electrons (spatial quaternion only), associativity holds. We can isolate an electron, put it in a trap, measure its charge and magnetic moment to twelve decimal places. No problem.

For quarks (both quaternions active), associativity fails. A single quark cannot be measured independently of its colour environment. This is not a practical limitation—it is algebraic.

6.2 Why Three Quarks Restore Associativity

But hadrons—protons, neutrons—are observed. How does a system of three quarks become associative when each quark alone is not?

The answer is colour neutrality. A proton contains three quarks, one red, one green, one blue. In the octonion algebra, the colour-vector components of the three quarks are:

$$\textit{Quark 1:}\ r_1 \cdot e_5 \quad (\textit{red})$$

$$\textit{Quark 2:}\ g_2 \cdot e_6 \quad (\textit{green})$$

$$\textit{Quark 3:}\ b_3 \cdot e_7 \quad (\textit{blue})$$

The composite colour-vector is:

$$C\_proton = r_1 \cdot e_5 + g_2 \cdot e_6 + b_3 \cdot e_7 \qquad (10)$$

For colour neutrality ($r = g = b$ in magnitude), the colour vector transforms under SU(3) as a singlet—the trivial representation. A singlet is invariant under all SU(3) transformations. And SU(3) is the associative part of the octonion automorphism group—it is precisely the subgroup that preserves the spatial quaternion $\{1, e_1, e_2, e_3\}$.

In other words: when the colour charges sum to a singlet, the non-associative cross-couplings between the two quaternions cancel. The proton's effective algebra collapses back to the spatial quaternion—four-dimensional, associative, observable. The colour quaternion is still there, but it is locked into a configuration that makes it invisible from outside.

This is exactly what confinement is. The quarks are not "trapped" by a confining potential as though they were balls on springs. They are algebraically invisible as individuals because the cross-coupling between the two quaternions is non-associative, and non-associative objects cannot produce unambiguous measurement results. Only colour-neutral combinations—where the non-associativity cancels—can be observed.

6.3 Mesons: The Two-Body Case

Mesons consist of a quark and an antiquark: $\bar{q}q$. The antiquark carries anti-colour: if the quark is red (e_5), the antiquark is anti-red ($-e_5$). Their colour vectors cancel:

$$C\_meson = r \cdot e_5 + (-r) \cdot e_5 = 0 \qquad (11)$$

Zero colour vector. The cross-coupling vanishes. The meson is associative—hence observable. It is unstable (the quark and antiquark can annihilate), but while it exists, it behaves as a purely spatial quaternion. You can measure its mass, spin, parity, charge. Every meson in the particle data tables is colour-neutral, exactly as the algebra requires.

But notice: the colour scalar e_4 component (the confinement energy) does not cancel in the meson—it adds. The meson's mass comes from E_conf, just as the proton's does. The colour vector vanishes (colour-neutral), but the colour scalar persists (mass).

7. The Wells Within Wells, Revisited

In [3], we introduced the image of wells within wells. With the two-quaternion framework, we can now be precise about this hierarchy:

Level	Algebra	Active Quaternions	Well Depth
Quarks in nucleon	𝕆 (Octonion)	Q_spatial + Q_colour	~1 GeV
Nucleons in nucleus	𝕆 → ℍ (residual)	Q_spatial + trace of Q_colour	~8 MeV/nucleon
Electrons in atom	ℍ (Quaternion)	Q_spatial only	~13.6 eV (H)
Atoms in molecule	ℍ (shared)	Q_spatial only	~1–10 eV
Molecules in crystal	ℍ (collective)	Q_spatial only	~0.01–1 eV

The pattern: as you go deeper into the well, the second quaternion wakes up. The electron lives entirely in Q_spatial. The quark lives in both. The transition from one to two active quaternions is the transition from electromagnetic to strong-force physics—from eV to GeV, from the Bohr radius to the proton radius, from hydrogen spectral lines to deep inelastic scattering.

8. A Worked Example: The Proton

Let us write down the proton explicitly. A proton contains two up quarks and one down quark, in a colour-neutral (singlet) state. We assign colours: up_1 = red, up_2 = green, down = blue.

8.1 Three Quark Octonions

Each quark is an octonion with both quaternions active. Using Eq. (6):

$$Q\_u1 = E\_u + L_1 \cdot e_1 + L_2 \cdot e_2 + S_3 \cdot e_3 + E\_conf \cdot e_4 + r \cdot e_5 + 0 \cdot e_6 + 0 \cdot e_7$$

$$Q\_u2 = E\_u + L_1' \cdot e_1 + L_2' \cdot e_2 + S_3' \cdot e_3 + E\_conf' \cdot e_4 + 0 \cdot e_5 + g \cdot e_6 + 0 \cdot e_7$$

$$Q\_d = E\_d + L_1'' \cdot e_1 + L_2'' \cdot e_2 + S_3'' \cdot e_3 + E\_conf'' \cdot e_4 + 0 \cdot e_5 + 0 \cdot e_6 + b \cdot e_7$$

8.2 The Colour Singlet

The colour-vector part of the proton's wave function is the antisymmetric combination:

$$\psi\_colour = (1/\sqrt{6}) \times (rgb - rbg + gbr - grb + brg - bgr) \qquad (12)$$

In octonion language, this antisymmetrization over e_5, e_6, e_7 ensures that the composite transforms as a singlet under SU(3). The non-associative cross-couplings between Q_spatial and Q_colour cancel pairwise—just as the k-components cancel in parahelium [3].

8.3 What the Proton Looks Like from Outside

From outside—from the electron's perspective—the proton is:

$$Q\_proton = (2E\_u + E\_d) + (total\ L) \cdot e_1 + \ldots + (E\_conf{,}total) \cdot e_4 + 0 \cdot e_5 + 0 \cdot e_6 + 0 \cdot e_7$$

The colour vector vanishes ($e_5 = e_6 = e_7 = 0$). But the colour scalar does not—E_conf persists in the e_4 component. This is the proton's mass! The 938 MeV that the electron "sees" is overwhelmingly confinement energy, sitting in the scalar of the colour quaternion. The electron orbits this mass without knowing where it comes from. It computes hydrogen spectral lines using Q_spatial alone, and gets every line right to 10 ppm.

The colour vector is hidden (confinement). The colour scalar is visible (mass). The proton shows its weight but hides its colour. This is the algebraic meaning of colour confinement: the vector part of the colour quaternion cancels to zero, but the scalar part survives as the proton's mass.

8.4 The Mass Puzzle

Can we compute the proton's mass from this framework? Not yet—not with pencil and paper. The proton mass arises from the dynamics of three interacting octonions confined to a colour-neutral state. In the standard model, this requires lattice QCD on supercomputers.

But the framework tells us where to look: the mass is E_conf, the scalar of the colour quaternion. It is the "energy" of the colour space, in precisely the same sense that E is the energy of physical space. Computing it requires solving the non-associative dynamics of Q_colour coupled to Q_spatial—the full octonion problem. We leave this to future work, but note that lattice QCD (which does compute the proton mass to 2%) is, in our framework, a numerical solution of exactly this octonion dynamics.

9. The Experiment: Proton–Electron Scattering Across Energy Scales

Everything in this paper has been algebra. Let us now make it experimental. Consider the simplest possible collision: a single electron hitting a single proton. By changing the electron's energy, we can watch the proton's algebraic nature change before our eyes—from one active quaternion to two—using nothing but a particle accelerator and a detector.

9.1 Regime I: eV — The Quaternion Handshake

At 13.6 eV, the electron's de Broglie wavelength is about 53,000 fm. The proton radius is 0.84 fm. The ratio is 63,000 to 1. The electron sees a point—a spatial quaternion with E = 938.3 MeV, spin-1/2, charge +1. The colour quaternion is invisible.

The result is hydrogen: twenty-six lines, accurate to 10 parts per million. Q_spatial meets Q_spatial. Both are associative. The physics is governed entirely by $\{1, e_1, e_2, e_3\}$—four dimensions, four degrees of freedom.

$$\lambda\_dB \approx 53{,}000\ fm \gg r\_p = 0.84\ fm \quad \rightarrow \text{ Only Q\_spatial visible } \rightarrow \mathbb{H} \times \mathbb{H}$$

9.2 Regime II: keV to MeV — Still One Quaternion

Increase the energy. The wavelength shrinks, but slowly:

Energy	λ_dB	λ / r_p	Proton appears as
1 keV	38,800 fm	46,000	Point charge (Rutherford)
10 keV	12,200 fm	14,500	Point charge
100 keV	3,700 fm	4,400	Point charge (Mott)

1 MeV	870 fm	1,040	Point charge
10 MeV	118 fm	140	Point charge

Even at 10 MeV, the electron's wavelength is 140 times the proton radius. The scattering follows the Mott formula—a single-quaternion calculation—and gives the right answer. The colour quaternion remains hidden. Q_spatial is the whole story.

9.3 Regime III: GeV — The Second Quaternion Revealed

Now set the electron energy to 1.47 GeV. At this energy, the de Broglie wavelength equals the proton radius:

$$\lambda_{dB} = 2\pi\hbar c / pc = 2\pi \times 197.3 / 1474 = 0.84\ fm = r_p \qquad (13)$$

This is the threshold. Below it, the electron sees one quaternion. Above it, the electron's wavelength penetrates the proton and resolves the second quaternion—the colour quaternion. In 1968, SLAC fired electrons at 4–20 GeV into protons:

Energy	λ_dB	λ / r_p	What the electron sees
1.5 GeV	0.83 fm	0.98	THRESHOLD — Q_colour appears
4 GeV	0.31 fm	0.37	Point-like quarks (both Q active)
8 GeV	0.16 fm	0.18	Quarks + gluon momentum
20 GeV	0.062 fm	0.074	Full two-quaternion structure

The scattering pattern showed Bjorken scaling: the electron was hitting point-like objects inside the proton—objects with both spatial and colour quantum numbers. It found:

Two up quarks and one down quark, carrying about 54% of the proton's momentum. These are full octonions—both quaternions active, colour vector nonzero (e_5, e_6, e_7).

Gluons, carrying about 41% of the proton's momentum. In the two-quaternion framework, these are the cross-couplings between Q_spatial and Q_colour—the non-associative products that mix the two quaternions. They are not objects in the usual sense. They are the algebra coupling the two quaternions to each other. Forty-one percent of the proton's momentum is carried not by things but by the relationship between the two quaternions.

Sea quarks (virtual quark–antiquark pairs), carrying about 5%.

9.4 The Punchline

The electron never changes. It is the same spatial quaternion at 13.6 eV and at 20 GeV. What changes is the resolution. Below 1.47 GeV, only Q_spatial is visible. Above 1.47 GeV, Q_colour is revealed. The algebra does not change—the octonion was always there. What changes is whether the experiment can see one quaternion or two.

SLAC did not discover quarks. SLAC discovered the proton's second quaternion.

10. The Collision Ledger: Pair Creation as Octonion Arithmetic

"The vacuum can only give birth to twins—a thing and its exact mirror. Because zero, on every axis, equals zero."

— This paper

In the preceding sections we described the proton's internal structure and showed how deep inelastic scattering reveals the otherworldly companion—the colour quaternion hiding inside every quark. But we treated the collision qualitatively. Now we do the bookkeeping. We write the electron and the quark as explicit algebraic objects—a quaternion and an octonion—and track every component through the collision. Eight dimensions in, eight dimensions out. No exceptions.

10.1 The Electron as Quaternion

The electron has no colour. Its otherworldly companion is zero on all four axes. It is a pure spatial quaternion:

$$Q\_e = E\_e + q\_e \cdot e_1 + s\_e \cdot e_2 + L\_e \cdot e_3 \qquad (10)$$

For a 1.5 GeV electron aimed at a stationary proton:

$$Q\_e = 1500 + (-1) \cdot e_1 + (-\tfrac{1}{2}) \cdot e_2 + 0 \cdot e_3$$

The real part is the energy in MeV. The e_1 component is the electric charge (−1 for the electron). The e_2 component is the spin projection (−½). The e_3 component is the orbital angular momentum (zero for a beam particle). The colour components e_4 through e_7 are all zero—absent, not merely small. The electron has no access to the otherworldly companion.

10.2 The Quark as Octonion

The up quark sitting inside the proton is the full eight-dimensional object. Take the red up quark as our example:

$$O\_q = E\_q + q\_q \cdot e_1 + s\_q \cdot e_2 + L\_q \cdot e_3 + E\_conf \cdot e_4 + r \cdot e_5 + g \cdot e_6 + b \cdot e_7 \quad (11)$$

$$O\_q = 313 + (+2/3) \cdot e_1 + (+1/2) \cdot e_2 + 0 \cdot e_3 + 310 \cdot e_4 + 1 \cdot e_5 + 0 \cdot e_6 + 0 \cdot e_7$$

The energy, 313 MeV, is the constituent quark mass—one-third of the proton. The charge is +2/3. The spin is +1/2 (opposite to the electron's, as they approach). The confinement energy on e_4 is 310 MeV—the quark's share of the colour field that holds the proton together. And on e_5 sits the colour charge: red = 1.

Note what distinguishes this up quark from the other up quark in the proton: the second up quark sits on e_6 (green) instead of e_5 (red), and has opposite spin. They have the same mass, the same electric charge, the same flavour—but they are not the same octonion, because they live on different imaginary axes. This is the Pauli exclusion principle, now stated in eight dimensions instead of four: no two identical fermions may occupy the same octonion.

10.3 The Total Before: Eight Conservation Laws

The total state before the collision is the component-wise sum of the electron (quaternion) and the struck quark (octonion). Since the electron has zero colour components, it contributes nothing to e_4 through e_7:

Dimension	Axis	Electron	Quark (u, red)	Total before
Real	Energy	1500 MeV	313 MeV	1813 MeV
e_1	Charge	−1	+2/3	−1/3
e_2	Spin	−1/2	+1/2	0
e_3	Orbital L	0	0	0
e_4	Confinement E	0	310 MeV	310 MeV
e_5	Red	0	1	1
e_6	Green	0	0	0
e_7	Blue	0	0	0

These eight numbers are the eight conservation laws. Every one must be identical in the total after the collision. Energy is conserved. Charge is conserved. Spin is conserved (or traded for orbital angular momentum within the spatial quaternion). Confinement energy is conserved as part of the total energy budget. And colour is conserved absolutely—no process can create or destroy net colour.

10.4 The Collision: Breaking the Eggshell

At 1.5 GeV, the electron's de Broglie wavelength is 0.83 fm—just under the proton radius of 0.84 fm. The eggshell cracks. The electron punches through the proton's exterior and hits one quark. It does not hit the proton as a whole; it hits one red up quark.

The electron transfers energy and momentum via a virtual photon. The photon is a quaternion object—it carries energy (real axis) and angular momentum (e_2, e_3) but no charge ($e_1 = 0$) and no colour (e_4 through $e_7 = 0$). The photon is the messenger of the spatial quaternion. It literally cannot touch the colour components.

After the photon exchange, the electron limps away with less energy. Say it retains 500 MeV and its spin has flipped from −1/2 to +1/2. The remaining 1000 MeV has been dumped into the quark system.

The struck quark recoils violently. It begins to fly away from the two spectator quarks. As it does, the colour field—the confinement energy on e_4—stretches like a rubber band. But this rubber band has a property unique to the strong force: its tension does not decrease with distance. It stays constant, at approximately 1 GeV per femtometre. Energy pours from the kinetic (real axis) into the confinement (e_4 axis). The e_4 component grows and grows.

10.5 The Birth of Twins: Pair Creation as Rotation

When the confinement energy on e_4 exceeds the threshold for creating a quark–antiquark pair (roughly 300 MeV for up/down quarks), the field snaps. Energy that was sitting on the real axis, accumulated on e_4, now rotates into the imaginary colour directions. A quark appears on $+e_5$ and an antiquark appears on $-e_5$. The vacuum has given birth to twins.

This is not a metaphor. In our algebra, pair creation is a rotation from the scalar (real or e_4) axis into a conjugate pair of imaginary directions. The rotation must produce equal and opposite imaginary components—otherwise the net colour would change, violating conservation on e_5. The only way to add something to the universe without changing any conserved quantity is to add a thing and its exact mirror: +1 and −1, +1/2 and −1/2, +1/3 and −1/3. On every imaginary axis, the twins sum to zero.

This is why pair creation always produces particle–antiparticle pairs. It is not a postulate of quantum field theory. It is a theorem of octonion arithmetic: zero equals zero on every axis.

10.6 The Four Particles After

Let us write the four outgoing particles explicitly. We take the simplest case: the colour field produces a down quark and a down antiquark. The antiquark (anti-red) pairs with the departing up quark (red) to form a colour-neutral meson ($\pi^+ = u + \bar{d}$). The new down quark (red) fills the colour hole in the remnant baryon.

	Real (E)	e_1 (q)	e_2 (s)	e_3 (L)	e_4 (E_c)	e_5 (r)	e_6 (g)	e_7 (b)
Electron (out)	E_e'	−1	+1/2	0	0	0	0	0
Struck u (in meson)	E_u'	+2/3	−1/2	0	E_m	+1	0	0
New d̄ (in meson)	E_d̄	+1/3	+1/2	0	E_m̄	−1	0	0
New d (in remnant)	E_d	−1/3	−1/2	0	E_r	+1	0	0

Now we check conservation on every axis:

Axis	Total before	Total after	Balance
Real (energy)	1813 MeV	E_e' + E_u' + E_d̄ + E_d = 1813	✔ (energy conservation)
e_1 (charge)	−1/3	−1 + 2/3 + 1/3 + (−1/3) = −1/3	✔
e_2 (spin)	0	+1/2 + (−1/2) + 1/2 + (−1/2) = 0	✔
e_3 (orbital L)	0	0 (redistributed freely within spatial Q)	✔
e_4 (confinement)	310 MeV	E_m + E_m̄ + E_r = 310	✔ (as total energy)
e_5 (red)	1	+1 + (−1) + 1 = 1	✔
e_6 (green)	0	0 + 0 + 0 = 0	✔
e_7 (blue)	0	0 + 0 + 0 = 0	✔

Every axis balances. The algebra closes. The twins—the d and d̄—contribute zero to every conserved quantity: their charges cancel (+1/3 and −1/3), their spins cancel (+1/2 and −1/2), their colours cancel (+1 and −1 on e_5). They came from zero and they sum to zero. The vacuum gave birth to a thing and its exact negative, and the universe noticed nothing.

10.7 Why the Twins Must Be Conjugate

This is a theorem, not an observation. Consider what would happen if the vacuum produced two quarks of the same colour—say both red. Then e_5 after = 1 + 1 + 1 = 3, but e_5 before = 1. The books would not balance. Colour conservation forbids it. The same argument applies to charge, spin, and every other imaginary axis.

The pair creation rule is therefore:

$$\textit{The vacuum can only create an octonion } O \textit{ and its conjugate } -O. \qquad (12)$$

Because $O + (-O) = 0$ on every imaginary axis.

This is the octonion version of the familiar rule that particle–antiparticle pairs are always produced together. But now we see why: the octonion has eight components, each independently conserved. The only way to change the system without changing any conserved quantity is to add zero in disguise—a pair of conjugates. Annihilation is the reverse: the conjugates collapse back onto the real axis, and out comes a photon—pure energy, pure scalar, zero on every imaginary axis.

10.8 What the Quark Is Afterward

The struck quark is not the same octonion after the collision. Its energy has changed (the real part increased by the photon's energy). Its spin may have flipped (the e_2 component changed). Its confinement energy is wildly different—the colour field was torn apart and rebuilt. The quark's identity—its charge on e_1 and its colour on e_5—is unchanged, because the photon cannot touch e_1 (it carries no charge) and cannot touch e_5 (it has no colour). But the quark is now bound in a meson instead of a baryon. Its life is completely different. Same charge, same colour—new address, new roommate, and a much shorter lease. The meson will decay in 10^{-8} seconds.

The proton, meanwhile, has lost one quark and gained another from the vacuum. The red hole was filled by the new d quark. The remnant—now containing u(green), d(blue), d(red)—is a neutron. The proton has become a neutron plus a pion. Conservation of charge confirms: proton (+1) = neutron (0) + π^+ (+1).

10.9 The Philosophical Implication

The otherworldly companion—the colour quaternion—is not visible. It is not directly measurable. No detector has ever registered a colour charge. And yet the algebra demands its existence: without three colour directions, two identical up quarks cannot coexist inside one proton without violating Pauli exclusion. Without a confinement energy scalar, the proton's mass has no explanation. Without the conjugate rule on all eight axes, pair creation has no mechanism.

The otherworldly companion is there because the algebra requires it—not because we chose to put it there, and not because a detector saw it. We allowed the algebra to lead, and it led us to a hidden quaternion that governs the strongest force in nature. The lesson is not about quarks. The lesson is about the courage to follow mathematics beyond what is directly visible, and to trust that what the algebra demands, nature provides.

11. Discussion

11.1 What We Have Shown

This paper makes seven claims:

First, the four normed division algebras ($\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$) correspond to four regimes of physics: classical mechanics, wave mechanics, quantum mechanics, and the strong force.

Second, the octonion is not a quaternion with extras. It is two quaternions—a spatial quaternion Q_spatial = (E, L_x, L_y, S_z) and a colour quaternion Q_colour = (E_conf, r, g, b)—coupled by the Cayley–Dickson multiplication rule. The colour quaternion has the same scalar-plus-three-vector architecture as the spatial quaternion. The scalar (e_4) is the confinement energy. The three-vector (e_5, e_6, e_7) is the colour charge.

Third, the symmetry group of the strong force, SU(3), arises as the stabilizer of the spatial quaternion inside the octonion automorphism group G_2. The eight gluons correspond to the eight generators of SU(3).

Fourth, colour confinement is a geometric necessity. The non-associativity of the octonion algebra arises from the cross-coupling between the two quaternions. Colour-neutral combinations cancel this cross-coupling, restoring associativity and hence observability.

Fifth, the proton's mass is the scalar of the colour quaternion—the confinement energy E_conf in the e_4 component. The colour vector cancels (confinement), but the colour scalar survives (mass). This is why 99% of the proton's mass is binding energy, not quark mass.

Sixth, the transition from one active quaternion to two is experimentally observable at $E \approx 1.47$ GeV—the threshold of deep inelastic scattering. SLAC discovered the proton's second quaternion in 1968.

Seventh, pair creation is an octonion theorem. The vacuum can only produce conjugate pairs—an octonion O and its negative −O—because each of the eight components is independently conserved, and the only way to add something to the universe without changing any conserved quantity is to add zero in disguise.

11.2 What We Have Not Shown

We have not computed the proton mass from first principles. In the standard model, this requires lattice QCD. In our framework, it requires solving the dynamics of Q_colour coupled to Q_spatial for three quarks in a colour-neutral state.

We have not addressed the electroweak force. The weak interaction involves SU(2), which is the automorphism group of the quaternions. This suggests that the weak force lives at the quaternion level. We defer this to a future paper.

We have not explained why there are three generations of quarks and leptons. The three division algebras $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$ applied over each other may produce three copies of the fermion spectrum. The number three—three colours, three generations, three spatial dimensions, three imaginary quaternion units—appears too often to be coincidental.

11.3 Falsifiability

A theory that explains everything explains nothing. What would falsify this framework:

1. Discovery of a fifth division algebra (Hurwitz says impossible).
2. Observation of a free quark (would break the associativity argument).
3. A hadron that is not a colour singlet.
4. A symmetry group of the strong force other than SU(3).

None of these have happened in fifty years of experimental particle physics.

12. Conclusion

"Hamilton needed three days to carve his equation into a bridge. Graves needed three months to find the octonions. Nature needed 13.8 billion years to build a physicist who could read both."

— This paper

We have climbed the Cayley–Dickson ladder from quaternions to octonions and found, at the top, not a foreign algebra but a familiar one—doubled. The octonion is two quaternions, face to face. The first governs space: energy, spin, angular momentum. The second governs colour: confinement energy, red, green, blue. The electron uses only the first. The quark uses both. The proton hides its second quaternion behind colour neutrality, showing only its mass—the scalar of the colour quaternion—to the outside world.

The pattern is now clear:

$\mathbb{R} \rightarrow$ *classical* | $\mathbb{C} \rightarrow$ *waves* | $\mathbb{H} \rightarrow$ *quantum* | $\mathbb{H}\times\mathbb{H} \rightarrow$ *strong force*

And the ladder stops at four, because mathematics has exactly four division algebras. The strong force is the last rung—not a new algebra, but the old algebra doubled.

In Paper 1, we showed that cosmological redshift is geometry. In Paper 2, that E = hf is geometry. In Paper 3, that the Pauli exclusion is geometry. In this paper, that colour confinement is geometry—the geometry of two quaternions coupled inside an octonion.

It is all one.

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