The Core of Metric D
Regular at the Centre, and What That Costs

The Near-Field Divergence Diagnosed in One Line, Repaired in One Function, Priced in Six Decades — and Caveat (i) Split in Two

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Abstract

Caveat (i) of the Allgemeine Feldtheorie records that metric D's source density near the observer exceeds the local inventory, and asks for a core regularization. This note supplies the diagnosis the caveat was missing, and finds that it names two different debts rather than one. The diagnosis is a single identity: for any static, spherically symmetric metric written in proper radial distance, the energy-density component of the Einstein tensor is $G^t_t = (2R\cdot R'' + R'^2 - 1)/R^2$, a function of the areal radius $R(r)$ alone — the lapse does not appear in it. Every observable the lapse carries, which is to say the redshift law, Tolman equilibrium and the temperature history, is therefore untouchable by any repair to the near field; the problem and its cure live entirely in one function. Expanding about the origin with $R = r + a\cdot r^2 + b\cdot r^3$ gives $G^t_t = 8a/r + (18b - 8a^2) + O(r)$, so a regular centre requires exactly $R''(0) = 0$. Metric D takes $R = r\cdot e^{-kr}$ and misses that condition by $-2k$, which is the whole disease: a curvature singularity sitting on the observer, who by the Copernican surrender is also the centre of coordinates. The repair follows from the condition rather than from taste: soften the radius inside the exponentials, $s(r) = \sqrt{r^2 + r_c^2} - r_c$, and write $\Phi = -k\cdot s$, $R = r\cdot e^{-k\cdot s}$. One function, one new constant. The centre becomes regular with $G^t_t(0) = -9k/r_c$; the exterior returns to metric D up to $e^{-2k\cdot r_c}$; and the redshift law, Tolman, the surface-brightness dimming and the angular turnaround at $z = e-1$ all survive by construction. The new constant is then bounded from below by solar-system dynamics and from above by the supernova Hubble diagram, and the bounds do not collide: the corridor runs from about 0.08 parsec to about 200 kiloparsec, six and a half decades wide. At a galactic 10 kpc the mass metric D demands inside Earth's orbit falls from 2.2×10^{-7} to 8×10^{-17} solar masses — ten orders below any ephemeris bound — for a cost of 0.0005 magnitudes at $z = 0.01$. What the repair does not do is flatten the profile: beyond r_c the source still runs as $1/r$, exceeding the cosmic mean by four orders at 1 Mpc and reaching order unity only near the horizon. Caveat (i) is therefore split: (i-a), the core, is closed here; (i-b), the profile, is opened, named, and is the larger debt. Everything is verified in the companion script. Every flag flown.

1. The Caveat, and What Was Missing From It

The foundations record the wound in one sentence: metric D's source density close to the observer exceeds the local inventory; a core regularization and a confrontation with solar-system dynamics are owed. What the sentence does not say is what, precisely, is divergent, why, or what a regularization would have to satisfy. Without that, 'a core regularization is owed' is a wish rather than a problem. This note turns it into a problem, solves the half that is solvable, and gives the other half its own name.

First the size of it, stated in the units a reader can check. Metric D's implied density at one astronomical unit is 2.09×10^{-11} kg/m^3, which is 2.4×10^{15} times the critical density, and the mass it demands inside Earth's orbit is 4.4×10^{23} kg — 2.2×10^{-7} solar masses, about a fifteenth of the Moon. Planetary ephemerides do not permit anything of that kind. This is not a rounding disagreement; it is a hard conflict with the best-measured dynamics in physics.

2. The Diagnosis: One Line, and the Lapse Is Not In It

Write the general static, spherically symmetric metric in proper radial distance — no gauge freedom left, two free functions:

$$ds^2 = -c^2\cdot e^{(2\Phi(r))}\cdot dt^2 + dr^2 + R(r)^2\cdot d\Omega^2$$

$\Phi(r)$ is the potential, whose exponential is the lapse $V = e^\Phi$; $R(r)$ is the areal radius, the function that says how large a sphere at coordinate r actually is. Compute the Einstein tensor and read off its energy-density component. It is:

$$G^t{}_t = (2\cdot R\cdot R'' + R'^2 - 1) / R^2 \qquad (1)$$

Look at what is not there. The lapse does not appear. The energy density of a static spherically symmetric spacetime is a functional of the areal radius alone.

This is the structural fact that makes the near-field problem tractable, and it deserves to be stated as a division of labour. The lapse carries the redshift — in a static spacetime $1 + z$ is nothing but the ratio of lapses — and with it Tolman equilibrium, the temperature history, and the whole of the series' spectroscopic evidence. The areal radius carries the source, and with it the angular observables. The two do not mix. So the near field can be repaired without putting a finger on any redshift observable, and no repair to the redshift observables could ever have fixed the near field. Half the difficulty of the caveat was not knowing which of the two functions was at fault.

3. The Condition: R″(0) = 0

Now expand about the observer. Any areal radius with a well-behaved centre begins as $R = r + a\cdot r^2 + b\cdot r^3 + \ldots$, with the leading r fixed by requiring circumferences to approach $2\pi r$ as spheres shrink. Substitute into Equation (1):

$$G^t_t = 8a/r + (18b - 8a^2) + O(r) \quad (2)$$

The divergence is entirely in the first term, and its coefficient is $8a = 4 \cdot R''(0)$. The condition for a regular centre is therefore not a matter of judgement:

$$R''(0) = 0 \quad (3)$$

Metric D takes $R = r \cdot e^{(-kr)} = r - k \cdot r^2 + \ldots$, so $a = -k$ and the density diverges as $-8k/r$. That is the disease, complete: metric D fails the regularity condition by exactly $-2k$, and the failure produces a curvature singularity at $r = 0$ — which, in a geometry whose whole point is that it is centred on the observer, is a singularity sitting on the observer. The Copernican surrender is priced in the foundations as a cost; this is a second instalment of that cost, and it was not priced.

4. The Repair: One Softened Radius

Condition (3) does not merely say that a repair exists; it says what a repair must do. Any function that vanishes to second order at the origin and returns to r outside some scale will serve. The simplest is the softened radius:

$$s(r) = \sqrt{(r^2 + r_c^2)} - r_c, \quad \Phi(r) = -k \cdot s(r), \quad R(r) = r \cdot e^{(-k \cdot s(r))} \quad (4)$$

One function, one new constant. Read it plainly: r_c is the radius inside which the geometry stops pretending to resolve structure. Because $s(0) = 0$ and $s'(0) = 0$, the substitution kills the offending term at the root — $R''(0) = 0$ exactly — and the centre becomes regular with a finite density:

$$G^t_t(0) = -9k / r_c$$

Outside, $s(r) \to r - r_c$, so both functions return to metric D, and the source approaches metric D's own value with the ratio $e^{(-2k \cdot r_c)}$ — which for any r_c in the corridor below differs from unity in the fifth decimal place. Nothing has been added to the theory except one length, and that length has a plain meaning: the scale below which a smooth cosmological metric was never entitled to be applied.

5. What Survives Untouched

By Equation (1), everything the lapse carries is safe automatically; it is worth listing the survivors explicitly, because the temptation with any repair is to fear that it has quietly broken something.

The redshift law. $1 + z = e^{(k \cdot s(r))}$, which is $e^{(kr)}$ for every r beyond the core — that is, for every source ever observed.

Tolman equilibrium and the temperature history. $T \cdot V$ = constant holds in every static spacetime whatever, so $T(z) = T_0(1 + z)$ is untouched — as the companion note Tolman from Staticity shows, it would be untouched by any change to R at all.

Tolman surface-brightness dimming. The $(1 + z)^{-4}$ law follows from Etherington reciprocity, which is independent of the metric's areal radius.

The angular turnaround. $dR/dr = 0$ now reads $k \cdot r^2/\sqrt{(r^2 + r_c^2)} = 1$, which for $r \gg r_c$ is $kr = 1$: the turnaround stays at $1 + z = e$, $z = e - 1 = 1.718$, displaced by a relative amount of order $(r_c \cdot k)^2$, which is 10^{-9}.

The supernova Hubble diagram, above the core. Luminosity distance is $D_L = R(r) \cdot (1 + z)^2 = r \cdot (1 + z)$ in both models — the same expression — so only the relation between r and z differs, and it differs only near the origin (§6).

6. The Corridor

The new constant must now be paid for, and the two bills come from opposite ends of the ladder.

From above: the supernovae. Inverting (4) gives $r = \sqrt{(s_z^2 + 2 \cdot s_z \cdot r_c)}$ with $s_z = (c/H) \cdot \ln(1 + z)$, so the softened luminosity distance exceeds metric D's by the factor $\sqrt{(1 + 2r_c/s_z)}$, and the magnitude residual is $\Delta m = 2.5 \cdot \log_{10}(1 + 2r_c/s_z)$. Requiring less than 0.01 magnitudes at the lowest redshift in the fit gives $r_c < 103$ kpc at $z = 0.005$, and $r_c < 205$ kpc at $z = 0.01$.

From below: the solar system. Inside the core the density is constant at $9k \cdot c^2/8\pi G \cdot r_c$, so the mass enclosed within radius r is $(3/2) \cdot k \cdot c^2 \cdot r^3/(G \cdot r_c)$ — falling as $1/r_c$. Requiring the mass inside Earth's orbit to stay under 10^{-11} solar masses gives $r_c > 0.08$ parsec; under 10^{-13} solar masses, $r_c > 8$ parsecs.

The bounds do not collide. The corridor runs from roughly a tenth of a parsec to a couple of hundred kiloparsecs — six and a half decades of room — and it contains, comfortably, exactly the scales a smooth cosmological metric ought to stop at: the galaxy, the halo, the group. Choose 10 kpc, a galactic radius, and the two ledgers read: the mass demanded inside Earth's orbit drops from 2.2×10^{-7} solar masses to 8×10^{-17}, ten orders of magnitude below any ephemeris bound; the cost at $z = 0.01$ is 0.0005 magnitudes, which no supernova campaign will ever see. The repair is, at galactic core radius, observationally free.

One caution, flagged at this document's standard: the ephemeris bound itself is quoted here parametrically rather than as a number, because the literature value has not been verified — caveat (ix) applies. The corridor's lower edge should be read as a formula awaiting its constant, not as a measured limit.

7. What Is Not Fixed: the Profile, and the Splitting of Caveat (i)

Now the part that does not resolve, stated as plainly as the part that does.

Softening caps the divergence; it does not flatten the profile. Beyond r_c the source still runs as $\rho \sim 1/r$, and measured against the cosmic mean that is an excess of 1.2×10^4 at 1 Mpc, 1.2×10^2 at 100 Mpc, and 2.7 at the Hubble radius. The excess only becomes order unity where the geometry ends. Equivalently, and more revealingly: a $1/r$ density is a mass function $M(<r) \propto r^2$, which is the profile of a halo centred on the observer, extending to the horizon.

This is a different debt from the one the caveat describes, and a larger one. The core singularity was a defect of the metric at a point; the profile is a claim about what fills the universe out to a gigaparsec. So caveat (i) should be split, and this note proposes the split:

(i-a) The core. Metric D's areal radius violates the regularity condition $R''(0) = 0$ by $-2k$, producing a $1/r$ curvature singularity on the observer and a solar-system conflict of ten orders. CLOSED: the condition is derived, a one-parameter repair satisfying it is built, every redshift observable is shown untouched, and the free constant is bounded into a six-decade corridor.

(i-b) The profile. Metric D's source is a $1/r$ halo centred on the observer, exceeding the cosmic mean by four orders at 1 Mpc. OPEN. It is not addressed here and is not addressed anywhere in the corpus. The natural place for it is the tension medium, whose active mass the string-cloud identification already sets to zero — so the question to put is whether the medium's own constitutive law delivers this profile or forbids it, which is a question for the perturbation theory named as the master calculation of the peaks campaign. Until that is answered, (i-b) stands as the cosmology's largest unaddressed observational exposure below the horizon scale.

8. Status, and What Is Owed

Established: Equation (1), and with it the separation of the lapse from the source; the regularity condition (3); that metric D violates it by $-2k$; that the violation is repairable by a one-parameter softening which leaves every lapse-carried observable exactly intact; and that the corridor for that parameter is non-empty by six and a half decades.

Not established, and not claimed: that (4) is the repair. Condition (3) is forced; the choice of $s(r)$ is not. Any function with $s(0) = s'(0) = 0$ returning to r outside the core will do, and different choices differ in the second decimal of the density profile inside r_c, where nothing is measured. A principled $s(r)$ — one derived rather than chosen, perhaps from the same seal reading that is owed for the areal radius itself — would be worth more than this one.

Also owed: the ephemeris constant (§6); and (i-b), which is now the front.

On authorship, at this series' first rule: the algebra of this note was performed and verified by machine, and it is a walkthrough to be checked rather than a result to be trusted. Equation (1) can be confirmed in an hour with any computer-algebra system, and Equations (2) and (3) follow from it by a series expansion a patient reader can do by hand.

9. The Sentence

The observer sat on a singularity of his own geometry, and the reason was one number — the second derivative of the areal radius, which metric D leaves at $-2k$ when regularity demands zero — so the cure is one softened length, bought for nothing at galactic scale, costing no redshift observable at all because the density never depended on the lapse in the first place; what remains, and what the caveat had hidden inside the same sentence, is the harder thing: that the source this geometry asks for is a halo centred on us and reaching to the horizon, and that debt is now to be paid in the medium, not in the metric.

References

The papers and notes of this series (the Allgemeine Feldtheorie — caveat (i), and §4a; Redshift as Infall / Metric D — the line element; De Sitter or Metric D; Tolman from Staticity — the lapse and what it carries; the Tension Medium series I–VI — the string-cloud constitutive law and the master calculation; The Family Law's Cosmic Rung). I. M. H. Etherington, Phil. Mag. 15, 761 (1933) — reciprocity, hence the surface-brightness law's independence of R. Verification script: Cosmology/metricD_core_check.py — Equation (1) derived symbolically for arbitrary Φ and R and cross-checked numerically against the full Einstein tensor; the expansion (2); the softened metric's regularity, far-field return and preserved turnaround; and the corridor table. (Citations from memory; the literature-verification pass applies, and applies with particular force to the ephemeris bound of §6.)

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