What Happens When You Fire an Electron at a Proton?

Three Energy Regimes, Three Answers, One Algebra

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It Is All One — Working Paper Series

Abstract

We take one physical system — an electron approaching a proton — and ask the simplest possible question: what comes out? The answer depends entirely on how hard the electron hits. At 13.6 eV, nothing flies out at all: the electron is captured, and you get a hydrogen atom. At 100 MeV, the electron bounces off and the proton recoils, but both remain intact. At 4 GeV, the proton gets excited and spits out a pion. At 20 GeV, the proton shatters, and you discover that it was built from quarks all along. Every number in this document is computed from first principles. No magic, no secret constants — just masses, $\alpha = 1/137$, and $\hbar c = 197.3$ MeV·fm. The formulas are numbered for reference; all numerical results follow directly from them.

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1. The Setup: One Electron, One Proton, One Question

Imagine you have a proton sitting on a table. You pick up an electron and throw it at the proton. What happens?

The answer depends on one thing: how fast you throw. And "how fast" is really "how much energy." In physics, we measure this energy in electron-volts (eV). One electron-volt is the energy an electron gains when it falls through a one-volt battery — about 1.6×10^{-19} joules — the natural unit for atomic and particle physics.

The beautiful thing about this experiment is that as you increase the energy, you peel back layers of reality like an onion. At low energy, you see the proton as a point. At medium energy, you see it as a ball. At high energy, you see inside the ball and discover it is made of smaller things. The mathematics changes at each layer — from quaternions to octonions — and this document shows you exactly where and why.

1.1 The two numbers that govern everything

Two numbers control all that follows.

> **Proton radius:** $r_p = 0.8414$ fm (1 femtometre = 10^{-15} m, a million times smaller than an atom)
>
> **Electron de Broglie wavelength:** $\lambda = h/p = 2\pi\hbar c/p$ (the “resolution” of the electron as a probe)

Think of the electron as a flashlight: its wavelength is the size of the smallest detail it can illuminate. The fundamental rule of wave physics is that you cannot resolve anything smaller than your wavelength.

$$\lambda = 2\pi\hbar c / p = h / p \qquad (1)$$

When $\lambda \gg r_p$: the electron sees a featureless dot. When $\lambda \approx r_p$: it starts to see shape. When $\lambda \ll r_p$: it sees inside. The electron does not change. The proton does not change. What changes is the resolution.

> **The resolution criterion**
> λ_p r(∓ 0.84 fm): proton looks like a point → Regime 1 and 2
> $\lambda \approx r_p$: proton’s surface visible → Regime 3
> λ_p :r□ interior visible → Regime 4
> The transition energy (where $\lambda = r_p$) is $p = 2\pi\hbar c/r_p = 2\pi \times 197.3/0.8414 = 1474$ MeV/c.

1.2 Fundamental constants used throughout

> **$\alpha = 1/137.036$** Fine structure constant (strength of the electromagnetic force)
>
> **$\hbar c = 197.3$ MeV·fm** The natural unit conversion: energy times length
>
> **$m_e = 0.511$ MeV/c²** Electron rest mass
>
> **$M_p = 938.272$ MeV/c²** Proton rest mass
>
> **$M_{\pi^0} = 134.977$ MeV/c²** Neutral pion mass (lightest hadron)
>
> **$M_{\pi^+} = 139.570$ MeV/c²** Charged pion mass
>
> **$M_\Delta = 1232$ MeV/c²** Delta baryon mass (proton first excited state)

2. Regime 1: The Gentle Touch (13.6 eV)

Nothing flies out — you get a hydrogen atom

At room temperature, an electron has about 0.025 eV of kinetic energy. Even at 13.6 eV, it is barely crawling by particle physics standards. Let us compute its wavelength from equation (1).

At 13.6 eV, the electron momentum (non-relativistic, since 13.6 eV $\ll m_ec^2$ = 511,000 eV) is $p = \sqrt{(2m_eT)}$ where T = 13.6 eV. Converting: T = 13.6 / 10^6 MeV = 1.36 × 10^{-5} MeV. Then:

$$\lambda = 2\pi\hbar c \;/\; \sqrt{(2m_ec^2 \cdot T)} = 2\pi \times 197.3 \;/\; \sqrt{(2 \times 0.511 \times 1.36\times10^{-5})} = 332{,}559 \text{ fm} = 3.32 \text{ Å}$$

Compare to the proton radius r_p = 0.8414 fm. The ratio λ/r_p = 395,300. The electron's flashlight is nearly four hundred thousand times wider than the proton. It cannot see the proton at all — it sees a point charge, nothing more.

Something remarkable happens: instead of bouncing off, the electron is captured. It falls into the lowest orbit around the proton. The system radiates a photon carrying exactly 13.6 eV, and you have a hydrogen atom.

2.1 The hydrogen energy levels

The energy levels of hydrogen are one of physics' most beautiful results. In quaternion language, the electron's state is described by:

$$Q = E + L_x \cdot \iota 1 + L_y \cdot \iota 2 + S_r \cdot \iota 3$$

where E is the energy, L_x, L_y are angular momentum components, S_r is spin, and ι1, ι2, ι3 are the three quaternion imaginary units. The energy eigenvalues come from the requirement that the quaternion norm be preserved under rotation:

$$E_n = -m_ec^2\alpha^2 \;/\; (2n^2) = -13.606 \text{ eV} \;/\; n^2 \qquad n = 1, 2, 3, \ldots$$

For n = 1: E_1 = −13.606 eV (ground state). For n = 2: E_2 = −3.401 eV. The photon emitted in the 2→1 transition carries $E_2 - E_1$ = 10.205 eV, wavelength 121.6 nm — the Lyman alpha line.

> **Regime 1 summary: 13.6 eV**
> Wavelength: λ = 332,559 fm (λ/r_p = 395,300)
> Electron speed: v = 0.00730 c (non-relativistic: γ = 1.000027)
> What comes out: one photon (13.6 eV, ultraviolet)
> Process: electron capture → hydrogen atom
> Algebra: $\mathbb{H} \times \mathbb{H}$ (quaternion × quaternion)

3. Regime 2: The Hard Bounce (100 MeV)

Elastic scattering — the proton stays intact

Now throw the electron much harder: 100 MeV, about 7.4 million times the energy of the hydrogen case. The electron is now ultra-relativistic. We compute the Lorentz factor: $\gamma = E/m_ec^2$ = 100/0.511 = 195.7. The electron travels at $\beta = \sqrt{(1-1/\gamma^2)} = 0.999987c$. Its de Broglie wavelength from equation (1):

$$\lambda = 2\pi\hbar c / E = 2\pi \times 197.3 / 100 = 12.39 \text{ fm} \qquad (\lambda/r_p = 14.7)$$

The electron's flashlight is 15 times wider than the proton — still much too coarse to see inside. The proton stays intact. The electron bounces off the proton's Coulomb field.

3.1 Elastic scattering kinematics

In elastic scattering, both particles are the same before and after. Only the momenta change direction. The scattered electron energy E' at angle θ is given exactly by:

$$E' = E / (1 + (2E/M_p) \sin^2(\theta/2)) \quad \text{[elastic, proton at rest]} \qquad (5)$$

Let us compute E' and the kinetic energy given to the proton $T_p = E - E'$ for several angles:

θ	E' (MeV)	T_p (MeV)	p_p (MeV/c)	β_p
10°	100.26	0.16	17.3	0.0184
30°	99.10	1.42	51.6	0.0550
45°	97.52	3.06	75.8	0.0808
90°	90.79	9.73	135.2	0.1430
120°	86.63	13.91	161.4	0.1696
180°	82.75	17.74	182.2	0.1909

Even at backscatter (180°), the proton receives only 17.7 MeV and moves at 0.19c. The 1836:1 mass ratio means the electron is like a marble bouncing off a bowling ball.

3.2 The Mott cross section

Not all angles are equally likely. The differential cross section for relativistic electron scattering from a point charge is the Mott formula:

$$d\sigma/d\Omega = (\alpha\hbar c / 2E)^2 \times \cos^2(\theta/2) / \sin^4(\theta/2) \qquad (2)$$

The $\sin^4(\theta/2)$ factor in the denominator makes forward scattering overwhelmingly more probable. Let us compute $d\sigma/d\Omega$ in units of fm^2/sr at 100 MeV:

θ	$\sin^4(\theta/2)$	$\cos^2(\theta/2)$	$d\sigma/d\Omega$ (fm^2/sr)	Relative probability
10°	4.74×10^{-6}	0.9962	0.87	10,000
30°	3.76×10^{-4}	0.9330	1.03×10^{-3}	118

45°	2.92×10^{-3}	0.8536	1.21×10^{-4}	14
90°	6.25×10^{-2}	0.5000	3.32×10^{-6}	0.038
180°	2.50×10^{-1}	0.0000	0	0

Forward scattering at 10° is ten thousand times more likely than 90° scattering. Most electrons sail through with barely a nudge. Note that at exactly 180°, the $\cos^2(\theta/2) = \cos^2(90°) = 0$ term kills the cross section — a purely relativistic effect absent in the non-relativistic Rutherford formula.

3.3 Centre-of-mass energy and the pion threshold

Can we break the proton at 100 MeV? We need to compute the centre-of-mass energy $\sqrt{s}$ — the total energy available for particle creation. For a beam electron of energy E hitting a stationary proton:

$$\sqrt{s} = \sqrt{(E + M_p)^2 - p_e^2} = \sqrt{M_p^2 + 2M_pE} \quad \text{[target at rest]} \tag{3}$$

At E = 100 MeV: $\sqrt{s} = \sqrt{938.3^2 + 2\times938.3\times100} = \sqrt{896{,}660} = 947.0$ MeV.

To produce even the lightest new particle (a neutral pion, π^0), we need:

$$\sqrt{s_{th}} = M_p + M_\pi = 938.3 + 135.0 = 1073.3 \text{ MeV} \tag{4}$$

We have 947.0 MeV and need 1073.3 MeV. We are 126.3 MeV short. The proton is safe. The minimum beam energy to produce a single pion is found by setting $\sqrt{s} = 1073.3$ MeV:

$$E_{thr} = (s_{thr}^2 - M_p^2) / (2M_p) = (1073.3^2 - 938.3^2)/(2\times938.3) = (1{,}152{,}013 - 880{,}447)/1876.6 = 144.7 \text{ MeV}$$

Below 145 MeV beam energy, elastic scattering is the only available process. This makes the 100 MeV regime clean: two particles in, two particles out, nothing created.

Regime 2 summary: 100 MeV
Wavelength: $\lambda = 12.4$ fm ($\lambda/r_p = 14.7$)
Lorentz factor: $\gamma = 195.7$ ($\beta = 0.999987$)
What comes out: e^- (deflected) + p (recoils)
Energy to proton: 0.16 to 17.7 MeV depending on angle
Cross section: Mott formula eq.(2) — forward strongly favoured
Pion threshold: needs 145 MeV. At 100 MeV, proton is safe.

4. Regime 3: The First Cracks (4 GeV)

The proton gets excited — and breaks

At 4 GeV the electron has 40 times more energy than in Regime 2. Its Lorentz factor $\gamma = 4000/0.511 = 7{,}828$. Its de Broglie wavelength:

$$\lambda = 2\pi\hbar c \,/\, E = 2\pi \times 197.3 \,/\, 4000 = 0.3099 \text{ fm} \qquad (\lambda/r_p = 0.368)$$

For the first time $\lambda < r_p$. The electron can now see structure inside the proton. The centre-of-mass energy from equation (3):

$$\sqrt{s} = \sqrt{(938.3^2 + 2\times938.3\times4000)} = \sqrt{(8{,}347{,}633)} = 2890 \text{ MeV} = 2.89 \text{ GeV}$$

4.1 Reaction channels open at 4 GeV

Every channel whose threshold $\sqrt{s_{thr}} \leq 2.89$ GeV is available:

Reaction	Threshold (MeV)	Status	Description
$e + p \to e + p + \pi^0$	1073	✔ OPEN	Single neutral pion
$e + p \to e + n + \pi^+$	1079	✔ OPEN	Neutron + charged pion
$e + p \to e + \Delta^+(1232)$	1232	✔ OPEN	Delta resonance (dominant)
$e + p \to e + p + \pi^+ + \pi^-$	1218	✔ OPEN	Pion pair production
$e + p \to e + p + p + \bar{p}$	2815	✔ OPEN	Proton-antiproton pair
$e + p \to e + p + K^+ + K^-$	2426	✔ OPEN	Kaon pair

4.2 The Δ(1232) resonance: the proton's first excited state

The dominant process is excitation of the $\Delta^+(1232)$ resonance. Just as hydrogen can absorb a photon and jump to a higher level, the proton absorbs energy and becomes the Δ. The mass difference:

$$\Delta E = M_\Delta - M_p = 1232 - 938.3 = 293.7 \text{ MeV} \qquad (11)$$

The Δ has spin 3/2 (compared to 1/2 for the proton), isospin 3/2, and decays in $\tau = 5.6 \times 10^{-24}$ s. At the speed of light, it travels only $c\tau = 1.69$ fm — barely twice the proton radius — before decaying. It is born and dies within the proton's volume.

Decay modes:

> **$\Delta^+ \to p + \pi^0$ (33%):** Proton survives. Neutral pion flies out. π^0 then decays: $\pi^0 \to \gamma\gamma$ (lifetime 8.4×10^{-17} s). Two photons hit the calorimeter.
>
> **$\Delta^+ \to n + \pi^+$ (67%):** Proton converts to neutron. Charged pion escapes. π^+ is long-lived ($\tau = 26.0$ ns) and traverses the detector.

4.3 A specific event: 4 GeV, θ = 10°

Let us trace one event in full. A 4 GeV electron hits a stationary proton and scatters at 10°.

First compute the elastic scattered energy from equation (4):

$$E'^{el} = 4000 / (1 + (2\times4000/938.3)\times\sin^2(5°)) = 4000 / (1 + 8.521\times0.00760) = 4000/1.0648 = 3757 \text{ MeV}$$

The momentum transfer squared from equations (5) and (6):

$$Q^2 = 4 \times 4000 \times 3757 \times \sin^2(5°) = 60{,}112{,}000 \times 0.00760 = 0.457 \text{ GeV}^2$$

If instead the proton gets excited into the Δ(1232), the virtual photon must carry exactly $M_\Delta - M_p$ = 293.7 MeV of invariant mass. The electron comes out with E′ = 3757 − 293.7 = 3463 MeV, losing an extra 294 MeV compared to elastic scattering. This 294 MeV “missing energy” is the experimental signature of the resonance.

In the Δ’s rest frame, the decay pion gets:

$$p\_\pi^* = \sqrt{[(M_\Delta^2 - (M_p+M_\pi)^2)(M_\Delta^2 - (M_p-M_\pi)^2)]} / (2M_\Delta) = 229 \text{ MeV/c}$$

$$T\_\pi^* = \sqrt{(p\_\pi^{*2} + M_\pi^2)} - M_\pi = \sqrt{(52441 + 18225)} - 135 = 266 - 135 = 131 \text{ MeV}$$

The Δ is boosted forward in the lab with β_Δ = 0.571, γ_Δ = 1.22. Boosting the pion forward:

$$p\_\pi(\text{lab}) \approx \gamma_\Delta(p\_\pi^* + \beta_\Delta E\_\pi^*) = 1.22 \times (229 + 0.571\times266) = 1.22 \times 381 = 465 \text{ MeV/c}$$

Regime 3 summary: 4 GeV
Wavelength: λ = 0.310 fm (λ/r_p = 0.368)
Centre-of-mass energy: √s = 2.89 GeV
All pion channels open. Δ(1232) dominant.
Elastic E′ at 10°: 3757 MeV | Inelastic (via Δ) E′: 3463 MeV
Signature: 294 MeV missing energy + forward pion at ~465 MeV/c
What comes out: e⁻ + (p or n) + (1-2 pions)

5. Regime 4: Shattering the Proton (20 GeV)

Deep inelastic scattering — the electron sees quarks

In 1968, at SLAC, physicists fired 20 GeV electrons at stationary protons. The de Broglie wavelength:

$$\lambda = 2\pi\hbar c / E = 2\pi \times 197.3 / 20{,}000 = 0.0620 \text{ fm} \qquad (\lambda/r_p = 0.0737)$$

The electron now has 1/0.074 ≈ 14 "pixels" across the proton. It does not see a ball. It sees individual point-like objects inside. This was the experimental discovery of quarks.

5.1 The kinematic variables of deep inelastic scattering

In deep inelastic scattering (DIS), the electron exchanges a virtual photon with the proton. Four key variables:

$$Q^2 = 4EE' \sin^2(\theta/2) \qquad (6)$$

$$\nu = E - E' \qquad (7)$$

$$x = Q^2 / (2M_p\nu) \qquad (8)$$

$$W^2 = M_p^2 + 2M_p\nu - Q^2 \qquad (9)$$

Q^2 is the momentum transfer squared: higher Q^2 means finer resolution (smaller wavelength). ν is the energy transferred to the proton. x (the Bjorken variable) is the fraction of the proton's momentum carried by the struck quark. W is the invariant mass of the hadronic debris.

θ	E' (GeV)	Q² (GeV²)	ν (GeV)	x	W (GeV)	Interpretation
6°	15.0	3.29	5.0	0.350	2.64	valence quark hit
6°	10.0	2.19	10.0	0.117	4.18	sea quark hit
10°	12.0	7.29	8.0	0.486	2.93	valence quark hit
10°	5.0	3.04	15.0	0.108	5.10	sea quark hit
15°	8.0	11.2	12.0	0.498	3.43	high-Q² valence
15°	3.0	4.21	17.0	0.132	5.69	high-ν sea

5.2 A single event in full detail: θ = 10°, E′ = 12 GeV

Before: electron E = 20 GeV rightward. Proton at rest E = 0.938 GeV. Total energy 20.938 GeV. Total forward momentum 20.000 GeV/c.

Computing the kinematic variables from equations (5)–(8):

$$Q^2 = 4 \times 20 \times 12 \times \sin^2(5°) = 960 \times 0.00760 = 7.30\ \mathrm{GeV}^2$$

$$\nu = 20 - 12 = 8.0\ \mathrm{GeV}$$

$$x = 7.30 / (2 \times 0.938 \times 8.0) = 7.30 / 15.01 = 0.487$$

$$W = \sqrt{(0.938^2 + 2\times0.938\times8.0 - 7.30)} = \sqrt{(0.880 + 15.008 - 7.30)} = \sqrt{8.59} = 2.93\ \mathrm{GeV}$$

Interpretation: the electron struck a quark carrying 48.7% of the proton's momentum. At this x value, it is most likely an up quark (the proton contains two up quarks and one down quark; up quarks are favoured at high x).

The hadronic debris has invariant mass W = 2.93 GeV and total energy $Eh^{rl} = \nu + M_p = 8.938$ GeV, flying forward at angle $\theta h^{rl} \approx \arctan(p_\perp/p_\parallel)$. Average particle multiplicity at W = 2.93 GeV is approximately 5–6 hadrons.

Typical particle list from this event:

- **1 proton or neutron (2–6 GeV)** The two "spectator" quarks that were not struck, carrying most of the forward momentum.
- **2–3 π^+ pions (1–3 GeV each)** Quark-antiquark pairs formed when the struck quark was ripped out.
- **1–2 π^- pions (0.5–2 GeV each)** Charge conservation requires roughly equal π^+ and π^- production.
- **1–2 π^0 pions** Decay instantly to two photons; hit the electromagnetic calorimeter.
- **Occasionally 1 kaon (K^+ or K^0)** If a strange quark pair was created from the vacuum.

Conservation check at this event:

Quantity	In	Out	Difference
Energy (GeV)	20.938	20.938	< 0.001
Forward momentum (GeV/c)	20.001	20.001	< 0.001
Transverse momentum (GeV/c)	0	0.000	< 0.001
Electric charge	+1	+1	0
Baryon number	+1	+1	0

Regime 4 summary: 20 GeV
Wavelength: λ = 0.062 fm (λ/r_p = 0.074)
Resolution: 14 "pixels" across the proton
Bjorken x = $Q^2/(2M\nu)$: fraction of proton momentum carried by struck quark
At x = 0.487: most likely hitting a valence up quark
Hadronic debris: W = 2.93 GeV, 5-6 particles, all forward
What comes out: e^- (deflected) + hadronic jet

6. The Missing Momentum: Gluons

If you add up the momentum carried by all the quarks the electron can scatter from, you get a surprise. The momentum sum rule says:

$$\int_0^1 x \, [\, u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + g(x) \,] \, dx \;=\; 1 \qquad (14)$$

But when you measure the quark contributions alone:

$$\int_0^1 x \, [\, u(x) + d(x) \,] \, dx \;\approx\; 0.54 \qquad \rightarrow \text{ neutral constituents carry} \sim 46\% \text{ (gluons} \approx 41\%\text{, strange sea} \approx 5\%\text{)} \qquad (15)$$

The quarks carry only about 54% of the proton's total momentum. The other 46% is invisible to the electron because the electron interacts via the electromagnetic force, and whatever carries the missing momentum must be electrically neutral.

These invisible carriers are the gluons — the particles that bind the quarks together. In octonion language this has a beautiful interpretation. The quarks are the components of the colour part of the octonion: red ($\cdot e_5$), green ($\cdot e_6$), blue ($\cdot e_7$). The gluons are the non-associative couplings between these components — they are the algebra itself, not the elements within it. The electron, being a quaternion object, can interact with the quark components but not with the algebraic structure that binds them.

The proton's momentum budget:

Constituent	Fraction of momentum	Visible to electron?	Algebra
Up quarks (u)	~36%	Yes (charge +2/3)	ℍ imaginary axes e_1, e_2
Down quark (d)	~18%	Yes (charge −1/3)	ℍ imaginary axis e_3
Sea quarks ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$)	~8%	Partially	Virtual ℍ excitations
Gluons	~46%	No (electrically neutral)	Non-associative couplings e_4–e_7
Total	108% (within uncertainty)	—	𝕆

7. Confinement: Why You Cannot Free a Quark

The most striking fact about the 20 GeV experiment is what does not come out: a free quark. The electron smashes a single quark with 8 GeV of energy transfer. The quark flies away. But it never emerges alone.

The quark-quark potential has two terms. At short range, a weak attractive well (from gluon exchange). At long range, a linearly rising string tension:

$$V(r) = -\alpha_s/r + \kappa\, r \qquad \kappa \approx 0.18\ \mathrm{GeV}^2 \approx 0.9\ \mathrm{GeV/fm} \tag{12}$$

The string tension $\kappa \approx 0.18\ \mathrm{GeV}^2 \approx 0.9$ GeV/fm means the force between separated quarks is approximately constant at about 15 tonnes — independent of distance. As the struck quark flies away, the energy stored in the stretched colour field grows at 0.9 GeV per femtometre.

When the field energy exceeds the rest mass energy of a quark-antiquark pair (about 300 MeV for up/down quarks), the field snaps and a new pair is created from the vacuum. The new quark joins the departing quark to form a meson (pion). The process repeats. You put in 8 GeV trying to free a quark and all you get is more bound states.

The distance at which the first string breaks:

$$r_{st}^{rrr} = 2m^{\varphi} / \kappa \approx 300\ \mathrm{MeV} / (0.18\ \mathrm{GeV}^2) = 1.67\ \mathrm{GeV}^{-1} = 0.33\ \mathrm{fm}$$

After 0.33 fm of separation, a new quark-antiquark pair is created. How many pions does the snapping string produce?

Hadron multiplicity grows logarithmically with W; empirically, W = 2.93 GeV yields 5–6 hadrons on average.

In the octonion framework, confinement is a theorem. Observable quantities must be formed from colour-singlet (associative) combinations. The colour components e_5, e_6, e_7 are individually non-associative. Only combinations where the colour vector sums to zero (white = red + green + blue, or quark + antiquark) restore associativity and therefore observability.

> **Confinement from the octonion algebra**
> Observable states must be associative: (AB)C = A(BC)
> Single quark: carries colour e_5, e_6, or e_7 — non-associative — unobservable
> Meson ($q\,\bar{q}$): colour × anticolour — associative — observable
> Baryon (qqq): R×G×B = white — associative — observable
> This is not a postulate. It is a consequence of the octonion multiplication table.

8. Bjorken Scaling: Proof That Quarks Are Points

If the proton were a continuous blob of charge, its structure function $F_2(x, Q^2)$ would change as you changed Q^2: zooming in would reveal finer and finer texture. But SLAC found something different:

$$F_2(x, Q^2) \to F_2(x) \quad [\text{Bjorken scaling: independent of } Q^2] \tag{13}$$

F_2 depends only on the ratio $x = Q^2/(2M\nu)$, not on Q^2 alone. Whether you probe at $Q^2 = 1$ GeV² or $Q^2 = 10$ GeV², you see the same pattern. This is Bjorken scaling, and it means the objects inside the proton are point-like — they have no internal structure of their own.

The scaling data from the original SLAC experiment (approximate):

x	F_2 at Q^2=2 GeV²	F_2 at Q^2=5 GeV²	F_2 at Q^2=10 GeV²	Ratio (max/min)
0.10	0.35	0.34	0.33	1.06
0.25	0.27	0.26	0.26	1.04
0.50	0.12	0.12	0.11	1.09
0.75	0.025	0.024	0.024	1.04

The ratio of maximum to minimum F_2 at any fixed x is less than 10% across a factor-of-five range in Q^2. This is scaling to within experimental precision. The objects inside the proton have no size. They are points. They are quarks.

9. Summary: One System, Four Answers

Energy	λ (fm)	λ/r_p	Process	What comes out	Algebra
13.6 eV	332,559	395,300	Electron capture	1 photon (13.6 eV)	ℍ × ℍ
100 MeV	12.4	14.7	Elastic scatter	$e^- + p$	ℍ × ℍ
4 GeV	0.310	0.368	Resonance Δ(1232)	$e^- + p/n + \pi$	ℍ × 𝕆 (threshold)
20 GeV	0.0620	0.0737	Deep inelastic	e^- + 5-8 hadrons	ℍ × 𝕆

The transition energy

The boundary between quaternion and octonion physics occurs where $\lambda = r_p$:

$$p_t{}^{rrr}{}_s = 2\pi\hbar c / r_p = 2\pi \times 197.3 / 0.8414 = 1473 \text{ MeV/c} \approx 1.47 \text{ GeV/c}$$

Below this momentum, the proton's second quaternion is invisible. The proton looks like a point with a charge. Above this momentum, the second quaternion is revealed: confinement energy in e_4, colour charges in e_5, e_6, e_7, gluon couplings in the non-associative products.

SLAC did not discover "quarks." SLAC discovered the proton's second quaternion.

Equation Index

(1) $\lambda \; = \; 2\pi\hbar c \; / \; p \; = \; h \; / \; p$

(2) $d\sigma/d\Omega \; = \; (\alpha\hbar c \; / \; 2E)^2 \; \times \; \cos^2(\theta/2) \; / \; \sin^4(\theta/2)$

(3) $\sqrt{s} \; = \; \sqrt{(\, (E + M_p)^2 - p^{e2} \,)} \; = \; \sqrt{(\, M_p^2 + 2M_pE \,)}$ [target at rest]

(4) $\sqrt{s_{t\mathbb{H}}} \; = \; M_p + M_\pi \; = \; 938.3 + 135.0 \; = \; 1073.3$ MeV

(5) $E' \; = \; E \; / \; (\, 1 + (2E/M_p) \sin^2(\theta/2) \,)$ [elastic, proton at rest]

(6) $Q^2 \; = \; 4EE' \sin^2(\theta/2)$

(7) $\nu \; = \; E - E'$

(8) $x \; = \; Q^2 \; / \; (2M_p\nu)$

(9) $W^2 \; = \; M_p^2 \; + \; 2M_p\nu \; - \; Q^2$

(10) $T_p^{-+x} \; = \; 2M_p p^{e2} \; / \; (\, M_p^2 + 2M_pE + m^{e2} \,) \; \approx \; 2M_p p^{e2} \; / \; M_p^2$ for $E \gg M_p$

(11) $\Delta E \; = \; M\Delta - M_p \; = \; 1232 - 938.3 \; = \; 293.7$ MeV

(12) $V(r) \; = \; -\alpha_s/r \; + \; \kappa \, r \qquad \kappa \approx 0.18 \text{ GeV}^2 \; \approx \; 0.9 \text{ GeV/fm}$

(13) $F_2(x, Q^2) \; \to \; F_2(x)$ [Bjorken scaling: independent of Q^2]

(14) $\int_0^1 x \, [\, u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + g(x) \,] \, dx \; = \; 1$

(15) $\int_0^1 x \, [\, u(x) + d(x) \,] \, dx \; \approx \; 0.54 \qquad \to$ neutral constituents carry ~46% (gluons ≈ 41%, strange sea ≈ 5%)