Four Quanta of Action: The Proton Mass–Radius Identity

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Abstract. The Strongforce papers of this series carry two geometric chains: the cage chain $r_p = (4/3)\cdot\hbar c/\Lambda$ (Paper 5), and the melting chain $r_p \cdot k_B \cdot T_c = (2/3)\cdot\hbar c$ (Paper 4, Section 9.5). Multiplied against the mass sum rule $m_p \approx 3\Lambda$, the two chains compress into a single identity containing no scale of this series at all — only the proton and two CODATA constants:

$$\mathbf{m_p \cdot c \cdot r_p = 4\hbar}$$

Equivalently: the proton's charge radius is exactly four of its own reduced Compton wavelengths, $r_p = 4\cdot\hbar/(m_p\cdot c)$. Against the muonic-hydrogen radius the dimensionless coefficient is measured at **3.9983 ± 0.0019** (−0.9σ from 4); against CODATA-2018 at **4.0008 ± 0.0090** (+0.1σ). The identity is thirty times tighter than either chain that produced it, because the two chains' one-percent slacks cancel in the product — a cancellation this note records but does not yet explain. Three consequences follow if the identity is exact, including a zero-parameter melting temperature $k_B \cdot T_c = m_p/6 = 156.38$ MeV against the measured 156.5 ± 1.5. The falsifier is already under construction: any radius measurement pinning r_p more than three standard deviations from 0.841236 fm kills the identity, and the current muonic error bar is only 2.4 of those units wide. Eddington's ghost is named in Section 5. Every symbol is introduced before use.

1. The Identity

Three symbols. m_p is the proton mass, 938.27208816(29) MeV/c^2 — the best-weighed object in subatomic physics. r_p is the proton's charge radius — the size of the cage, measured by how electrons and muons scatter from it and orbit it. ħ is the quantum of action and c the speed of light; their product $\hbar c = 197.3269804$ MeV·fm is exact to the digits shown.

The claim, in three equivalent costumes:

$\mathbf{m_p \cdot c \cdot r_p = 4\hbar}$ — the proton is four quanta of action.

$\mathbf{r_p = 4 \cdot \bar{\lambda}_p}$ — the cage is four of the proton's own reduced Compton wavelengths ($\bar{\lambda}_p = \hbar/m_p \cdot c = 0.2103089$ fm).

$\mathbf{m_p = 4\hbar c / r_p}$ — the mass is four door-prices of the room it lives in.

The prediction is parameter-free and essentially exact on the theory side: $r_p = 4\cdot\bar{\lambda}_p =$ **0.8412356 fm**, with an uncertainty inherited from m_p of order 10^{-10} fm — a million times below any measurement. Everything therefore rests on the measured radius.

2. The Audit

The dimensionless coefficient $m_p\cdot c\cdot r_p/\hbar$, measured four ways:

dataset	r_p (fm)	coefficient	σ from 4
muonic hydrogen (Antognini et al. 2013)	0.84087(39)	**3.9983 ± 0.0019**	−0.94
CODATA 2018	0.8414(19)	**4.0008 ± 0.0090**	+0.09
PRad ep scattering (2019)	0.831(14)	3.951 ± 0.067	−0.73
old H spectroscopy (CODATA-2014)	0.8751(61)	4.161 ± 0.029	**+5.6**

As a product: $m_p\cdot c^2\cdot r_p = 788.96 \pm 0.37$ MeV·fm (muonic) against $4\hbar c = 789.31$ — a deviation of 0.04%.

Two readings of this table. First, the identity holds at the one-sigma level against both modern determinations — a five-character formula landing inside a 0.05% error bar. Second, and historically pointed: the identity takes a side in the proton-radius puzzle. The old spectroscopic radius of 0.875 fm — the CODATA value for a decade — is excluded at 5.6σ. Had this note been written in 2012, the identity would have stood as a prediction that the accepted radius was wrong by four percent; the muonic measurement then moved the world's radius onto the identity's line. That is retrodiction, not prediction, and is recorded as such — but it is the direction a real identity moves.

3. Where the 4 Comes From in This Series

The identity is the product of the Strongforce chains, and the 4 factorizes as **3 × (4/3)**:

- **The 3:** the mass sum rule of Paper 5 — a hadron weighs its confinement units, $m_p = \Lambda\cdot\Sigma(e_0+e_4) \approx 3\Lambda$. One confined quark weighs one third of the proton; the quark's Compton cell is therefore $\hbar c/\Lambda = 3\cdot\bar{\lambda}_p$ — the seven-sphere radius of the funnel.
- **The 4/3:** the tetrahedral dressing of Paper 5 — the charge radius is the seven-sphere radius dressed by the ratio of the sphere to the inscribed tetrahedron's circumsphere, $r_p = (4/3)\cdot\hbar c/\Lambda$.

Chain them: $r_p = (4/3)\cdot 3\cdot\bar{\lambda}_p = 4\cdot\bar{\lambda}_p$. The four is three actors times one stage geometry.

The cancellation, recorded honestly. The chains individually carry one-percent slack: the presets give $m_p = 3.0267\cdot\Lambda$ (not 3Λ), and $(4/3)\cdot\hbar c/\Lambda = 0.8487$ fm against the measured 0.841 (+0.9%). Run the chains as the presets state them and the product is $4.036\cdot\hbar$ — off by 0.9%. Run the identity against the sky and the product is 4.000 to a tenth of a percent. **The two slacks cancel, and the identity is sharper than the machinery that suggested it.**

Either the cancellation is accidental at the one-percent level — possible, and the quarantine column exists for exactly this — or the exact statement is $m_p \cdot c \cdot r_p = 4\hbar$ and the preset chain is the approximation, in which case the presets' e_0 corrections and the tetrahedral factor are two faces of one constraint still owed a derivation. This note claims the observation, not the theorem.

4. Consequences If Exact

(i) The melting temperature loses its last input. Combining with Paper 4's chain $r_p \cdot k_B \cdot T_c = (2/3) \cdot \hbar c$, the radius drops out entirely:

$k_B \cdot T_c = m_p/6 =$ **156.379 MeV** $= 1.815 \times 10^{12}$ K

against the heavy-ion freeze-out measurement 156.5 ± 1.5 MeV (0.08σ). The melting temperature of matter, from the proton mass alone: the cage opens at one sixth of the weight it guards.

(ii) The family law wears the proton's own clock. The colour rung's thermal cell becomes $R = r_p \cdot 3/(4\pi) = 0.20083$ fm — verified identical to $\hbar c/(2\pi \cdot k_B \cdot T_c)$ — so the rung sits on the $T \cdot R = 0.3644$ mm·K line with the proton supplying both coordinates.

(iii) A one-line statement of confinement's price. In the winding language of this series, mass is a winding rate ($\nu = mc^2/h$) and radius is a room. The identity says the proton's room is exactly four beats of its own clock across — the cage is commensurate with the dance it contains. Whether that commensurability is a closure condition (four windings of the confined state fitting the cage, in the manner of the discrete-orbits note) is the natural conjecture, and it is left as one.

5. What This Is Not

Not dimensional analysis vindicated. Any confinement theory puts $m_p \cdot c \cdot r_p$ at a few $\hbar$; lattice QCD generates both numbers from Λ_{QCD}, so a coefficient of order unity is guaranteed. The content here is not the scale but the coefficient being an **integer** to a part in a thousand, and that integer factoring as the series' own $3 \times 4/3$.

Eddington's ghost, named. A single dimensionless coincidence involving small integers is exactly the species of claim that has embarrassed physics before (137, the large-number hypotheses). The defenses available: the coefficient is measured at the 0.05% level, not the ten-percent level where numerology thrives; it decomposes along a chain this series built for independent reasons (the mass sum rule and the tetrahedral factor precede this note); and it carries a near-term falsifier (Section 6). The prosecution's case: the 3 and the 4/3 each hold only at the percent level, and their conspiracy to a tenth of a percent is unexplained. Both columns are printed.

Literature pass owed, emphatically. The observation $r_p \approx 4 \cdot \bar{\lambda}_p$ is simple enough that someone may own it — holographic QCD, skyrmion phenomenology, and bag-model literatures all traffic in $m_p \cdot r_p$ relations. Priority is claimed nowhere until the pass this corpus owes is done.

6. Falsifiers

The identity predicts $r_p = 0.8412356$ fm with no adjustable anything. Therefore:

- **Any radius determination more than 3σ from 0.84124 fm kills it.** The muonic error bar (0.00039 fm) is already only 2.4 such units wide; the identity currently sits at −0.9σ inside it. CREMA-class muonic spectroscopy, MUSE (muon scattering), and PRad-II all aim at few-$\times 10^{-4}$ fm precision. A converged world radius at, e.g., 0.8405(2) fm ends this note at 3.7σ; a convergence onto 0.8412 crowns it.
- **The melting-temperature cross-check:** if exact, $k_B \cdot T_c = 156.38$ MeV. A future freeze-out or lattice determination at, e.g., 158.5 ± 0.5 MeV excludes the combined chain at 4σ.
- The identity dies with either; it cannot be retuned, because it has nothing to tune.

References

CODATA 2018 (E. Tiesinga et al., Rev. Mod. Phys. 93, 025010, 2021); A. Antognini et al., Science 339, 417 (2013) — muonic hydrogen; R. Pohl et al., Nature 466, 213 (2010); W. Xiong et al. (PRad), Nature 575, 147 (2019); Particle Data Group, Phys. Rev. D 110, 030001 (2024); A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561, 321 (2018) — freeze-out temperature; and the papers of this series (Neutron Decay as Octonion Algebra — the mass sum rule and the 4/3; The Strong Force as Geometric Necessity §9.5 — the melting chain; The Radii of the Worlds; the Postulates). Verification script: four_quanta.py. *(Citations from memory; the literature-verification pass — caveat (ix) of the foundations paper — applies to every one.)*

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