The Quantum Leap as a Two-Level Rotation
E = hf as the Beat of Two Clocks

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Abstract

Bohr's quantum leap has been a scandal since 1913: the electron changes level and emits a photon, but the formalism says nothing about what happens *during* the change. This note gives the leap a continuous, geometric picture — the picture the series always wanted — and gets it right by placing the rotation on the correct object. A single energy eigenstate does not, by itself, leap: its time evolution $e^{-iEt/\hbar}$ is a **global phase**, which leaves the physical ray untouched and is therefore unobservable. What rotates, during a real transition, is the **two-level state** — the superposition of the initial and final levels — turning on the Bloch sphere of the subspace $\{|1s\rangle, |2p\rangle\}$ under the drive of the electromagnetic field. Each eigenstate is a clock, ticking at its own rate $\omega_i = E_i/\hbar$ (the Compton clock this series already carries in the Tick Count and Schrödinger notes); the transition is the two clocks *beating* against each other. And the light is the beat: the atom's radiating dipole, in the superposition $\alpha|1s\rangle + \beta|2p\rangle$, oscillates at the **difference** frequency $(E_{2p} - E_{1s})/\hbar$ — so the photon's frequency is the beat of the two clocks, and E = hf is a geometric identity after all, but a beat of two clocks rather than the spin of one. For Lyman-α the arithmetic is exact: ΔE = 10.20 eV gives λ = 121.5 nm against the measured 121.567 nm (quantum_leap_check.py). The leap's *duration* is then physical and definite — the Rabi period when the field drives it, the 2p spontaneous-emission lifetime (~1.6 ns) when it proceeds freely — both fixed by the dipole matrix element, not read as a parameter-free geometric time. The 'transfigured' instant the leap passes through, purely quantum and observable in no basis, is the equal superposition on the Bloch equator ($|\alpha| = |\beta|$), a true and vivid picture of the midpoint. What the series may claim is therefore real and clean: the leap **is** a rotation (of the two-level state), E = hf **is** geometric (the beat of two Compton clocks), and the photon **does** speak both languages (energy on the real axis, angular momentum on the imaginary). Standard quantum mechanics supplies the transition probabilities and selection rules unchanged; this note supplies only the reading. Every symbol is introduced before use.

1. The Scandal, and the Right Geometric Answer

In 1913 Bohr placed electrons on discrete levels and had them jump between, emitting light of frequency $f = \Delta E/h$. The prediction was triumphant and the mechanism outrageous: where is the electron mid-jump? Schrödinger hated it — 'if all this damned quantum jumping were really here to stay, I should be sorry I ever got involved.' The honest geometric answer is that there is no instant jump; there is a continuous rotation. The only question is *what rotates*. The mistake to avoid is rotating a single level. The right object is the two-level state.

2. Two Clocks

Give each level its quaternion label — energy on the real axis, angular momenta on the imaginary — and read its time evolution. An eigenstate of energy E evolves as

$$|\psi(t)\rangle = e^{-iEt/\hbar}\, |\psi\rangle,$$

a steady rotation of phase at rate $\omega = E/\hbar$: a **clock**, and precisely the Compton clock the Tick Count note counts and the Schrödinger note factors out. But one clock, alone, shows nothing: $e^{-iEt/\hbar}$ is a global phase, and the physical ray $|\psi\rangle\langle\psi|$ is untouched by it. A single level's rotation is invisible — and a half-turn, $|\psi\rangle \to -|\psi\rangle$, lands on the *same ray*. So no single level can leap: 1s and 2p are different rays, and a level's own rotation never leaves its ray. The physics of the leap is not in one clock; it is in the *relation* of two.

3. The Leap Is a Two-Level Rotation

A transition is a rotation in the two-dimensional subspace spanned by the levels it connects — the Bloch sphere of a two-state system — driven by the electromagnetic field (the Rabi rotation). During it the atom is a genuine superposition,

$$|\Psi(t)\rangle = \alpha(t)\, |1s\rangle + \beta(t)\, |2p\rangle,$$

with the state point travelling from the north pole (pure 1s) toward the south (pure 2p) as α and β trade weight. The midpoint $|\alpha| = |\beta|$ is the Bloch equator — the 'transfigured' instant, neither level, a coherent both-at-once that no energy measurement can catch mid-flight. This is a real rotation of a real state, and it is where the leap actually lives.

4. The Light Is the Beat

What radiates is the atom's electric dipole, and in the superposition it carries the **relative** phase of the two clocks:

$$\langle d\rangle(t) \propto \mathrm{Re}\left[\, \alpha^*\beta \cdot \langle 1s|\, d\, |2p\rangle \cdot e^{-i(E_{2p} - E_{1s})t/\hbar} \,\right].$$

This oscillates at the **beat frequency** $\omega = (E_{2p} - E_{1s})/\hbar$ — the *difference* of the two clocks' rates — and an oscillating dipole radiates at its own frequency. The photon is that radiation; its frequency is the beat. Planck's relation is thereby geometric, exactly as hoped, and exactly stated:

Quantity	Value	What it is
$\Delta E = E_{2p} - E_{1s}$	10.204 eV	the difference of two clock energies
$\nu = \Delta E/h$	2.467×10^{15} Hz	the beat frequency of the superposition
$\lambda = c/\nu$	121.5 nm	Lyman-α, measured 121.567 nm

E = hf is the statement that the emitted light is the beat of the two Compton clocks — the Tick Count note's reading of mechanics as clock-counting, now applied to two levels at once. The common phase of the two clocks is unobservable (it is the global phase of §2); their *difference* is the line.

5. The Duration Is Physical

Because the leap is a driven rotation of a real state, its timescale is set by dynamics, not geometry. **Driven**, the transition takes a Rabi period $2\pi/\Omega_R$, with the Rabi frequency Ω_R proportional to the dipole matrix element $\langle 1s|d|2p\rangle$ times the field amplitude — a fast leap in a strong field, a slow one in a weak field. **Free**, the excited level decays with its spontaneous-emission lifetime — about 1.6 ns for 2p — fixed by the same matrix element through the Einstein A-coefficient. There is no parameter-free 'rotation time': the leap is as quick as the coupling makes it, and that is a measurable, standard quantity.

6. What Is Geometric, What Is Standard

Geometric (claimed). The leap is a rotation — of the two-level state, on its Bloch sphere. E = hf is a geometric identity — the beat of two Compton clocks. The photon carries energy on the real axis and angular momentum on the imaginary, and mediates the transition by exchanging both. These are the series' own readings, and they are correct.

Standard (used, not replaced). The transition *probability* (Fermi's golden rule), the *selection rules* (Wigner–Eckart), and the lifetimes (the Einstein coefficients) are ordinary quantum mechanics, unchanged. This note gives them the beat-of-clocks picture; it does not alter a number. The single-level rotation $\omega = E/\hbar$ is not the photon frequency, and no duration is read from it — those were the errors of the earlier draft, corrected here at the root by rotating the right object.

7. The Sentence

The leap was never one level turning — a clock whose ticking no one can see; it is two levels beating, the electron a coherent both-at-once on the equator between them, and the light that leaves is the beat, so E = hf is geometric exactly as Bohr's scandal always hinted, only it is the difference of two Compton clocks that sings, not the turning of one.

References

N. Bohr, Phil. Mag. 26, 1 (1913); M. Planck (1900); the Rabi two-level model, the Einstein A/B coefficients, Fermi's golden rule and the Wigner–Eckart theorem (standard quantum optics and quantum mechanics); and the notes of this series (the Tick Count — mechanics as clock-counting; Schrödinger from the Geometry — the shutter/Compton clock; the State Quaternion label). Verification script: quantum_leap_check.py (stdlib). This edition supersedes the earlier draft that rotated a single state quaternion from +1 to −1. (Citations from memory; the literature-verification pass applies.)

Acknowledgment: drafting and numerical checks by machine (Claude, Anthropic), in conversation; the instinct that the quantum leap is a continuous rotation and that $E = hf$ is geometric is the author's, and both are here placed on their correct footing — the rotation on the two-level Bloch sphere, and $E = hf$ as the beat of two clocks.