From Space Quaternion to Particle: Derrick's Wall, the Two Doors, and a Baryon Computed

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Abstract

This note executes, as far as it can honestly be executed today, the construction the series has pointed at since its first page: the transformation of the space quaternion into a particle. The program is stated as three steps and then run. Step one promotes the quaternion phase — the home of matter since Weyl's repair — to a field: at every point of space, a unit quaternion U(x), a point on the three-sphere. A particle is then a knot in this field: a configuration that winds, closes, and cannot be combed flat, its winding number an integer no smooth deformation can change. Step two meets the wall and names it: Derrick's theorem (1964), proved here in two lines, shows that the obvious energy — the gradient — makes every such knot collapse to a point. The theorem is identified as the mathematical form of this series' stiffness problem: the winding needs a second term, of opposite scaling, to catch the fall. Step three walks through the two known doors, and finds that they are exactly the two classes of the companion Floors of Condensation note: the lepton door (time-periodicity — a Q-ball-*type* idea, but one this note finds **empty**: $E = \hbar\omega Q$ at $\omega = \omega_C$, $Q = 1$ gives $E = mc^2$ only tautologically, because ω_C is *defined* as $mc^2/\hbar$, and no potential or conserved charge is exhibited to make it a real Q-ball; §4.1 says so at full mast) and the baryon door (topology plus a quartic stiffness — Skyrme's construction). The baryon door is then not merely described but **reproduced**: the hedgehog configuration is solved numerically by shooting, and the standard Skyrmion condenses in the calculation. It must be stated plainly that this is a *reproduction* of Skyrme's massless model under a *reinterpretation* (SU(2)-valued field read as the space-quaternion phase), not an independent derivation of the proton from the framework — the reading is an identification offered, not one forced by the geometry. The computed diagnostics: winding number $B = 1.0000$; the Derrick virial $E_2 = E_4$ satisfied to 0.2 percent; soliton energy $E = 36.47$ (F_π/e), reproducing the literature value and standing 1.2319 above the topological bound $3\pi^2$; with the Adkins–Nauenberg–Witten calibration, a classical core mass of 863 MeV and a half-angle radius of 0.49 fm — a nucleon-sized, nucleon-weight object built from nothing but a quaternion field and one stiffness constant. The nucleon's spin-half then arrives by the double cover: the Finkelstein–Rubinstein construction — already cited by this series' Pauli paper for exchange-as-rotation — permits quantizing the spinning knot as a fermion, and the N–Δ splitting is rotational energy: the mass is the dance, computed. Priority is stated

without reservation: the model is Skyrme's (1961), its solution Adkins, Nauenberg and Witten's (1983), its legitimacy Witten's large-N argument; this note's contribution is the reading — their SU(2)-valued field is the space-quaternion phase, their hedgehog is this series' hedgehog, their winding number is the trinity's topological floor, and their one fitted constant is the entire remaining debt of this framework, located and named. Four figures, all computed from the solution rather than sketched; every symbol introduced before use; every flag flown.

1. The Question, Stated as a Construction

The series has said from its foundations that matter is condensed space, that the condensation lives in the quaternion phase rather than the metric scale, and that the winding is the mass. Those are readings. The question this note faces is the constructive one: can the reading be made to *build something*? Given nothing but the space quaternion and an energy rule, can a configuration be exhibited — computed, not described — that has the mass, the size, the spin, and the permanence of a particle?

What 'transform the space quaternion into a particle' must mean, precisely: exhibit a field configuration of the quaternion phase whose energy is finite, whose extent is finite and stable (it neither spreads nor collapses), whose conserved labels (winding numbers) match a particle's quantum numbers, and whose calibrated energy lands at a measured mass. Anything less is metaphor. This note reaches that standard for one particle — the nucleon — and locates exactly what still stands between the framework and the electron.

The route was prepared by the companion notes. *The Delay That Makes G* showed the condensed-space reading is quantitative at the top of the ladder (the universe sits at its own horizon). *The Floors of Condensation* showed that condensation is everywhere floored, and classified matter into two classes — the singularity (lepton) and the trinity (baryon) — by the kind of floor that catches it. This note pours a floor: it shows, in a computed example, what catches the collapse and what the caught object weighs.

2. The Field: Space Wearing a Quaternion at Every Point

Step one of the construction. Take the phase of the space quaternion — the unit-quaternion part, the element of S^3 that the state turns through — and let it vary from place to place: a field U(x), assigning to every point of space a unit quaternion. This is the Lanczos–Conway road of the field-equations note ('the state quaternion becomes a field') taken literally, and it is the only promotion the framework needs.

Two structural facts make this field a particle factory. First, the target is a sphere: the unit quaternions form the three-dimensional sphere S^3. Second, the domain is effectively also a sphere: physical space, with the boundary condition that the field settles to the identity U = +1 far away, wraps up — all of infinity becomes a single point, and space becomes an S^3 as well. A configuration of the field is therefore a map from a three-sphere to a three-sphere, and such maps carry an integer that no continuous deformation can change: the number of times the domain wraps the target. In the trade this is $\pi_3(S^3) = \mathbb{Z}$; in this series' language it is the statement that *windings must close* — a map either wraps a whole number of times or it is not a map. The integer is the candidate baryon number. You can slide a knot along a rope; you cannot slide it off a rope with no ends.

The configuration this note computes is the *hedgehog* — the maximally symmetric knot, in which Hamilton's three units are combed radially outward:

$$U(x) = \cos f(r) + \sin f(r) \cdot (\hat{x} \cdot (i, j, k))$$

Read it slowly. At radius r, in the direction of the unit vector $\hat{x}$, the field is a unit quaternion whose real part is cos f(r) and whose imaginary part points along the *spatial* direction $\hat{x}$, translated into the quaternion units — internal axes locked to external directions. The single profile function f(r) carries everything. The boundary conditions $f(0) = \pi$, $f(\infty) = 0$ mean: at the core the quaternion is −1 (fully inverted, pure real), far away it is +1 (the identity, pure real), and in between it leans through the imaginary directions — the quills of Figure 1. That passage, from −1 through the imaginary sphere to +1, in every radial direction at once, wraps S^3 exactly once: winding number one, a single indivisible knot. The name is not this note's: the tension-medium papers call the radially-combed fall lines of the cosmos 'the hedgehog.' The same word, the same geometry, at two rungs — the cosmological hedgehog is combed in real space; the baryon's is combed in the quaternion fiber.

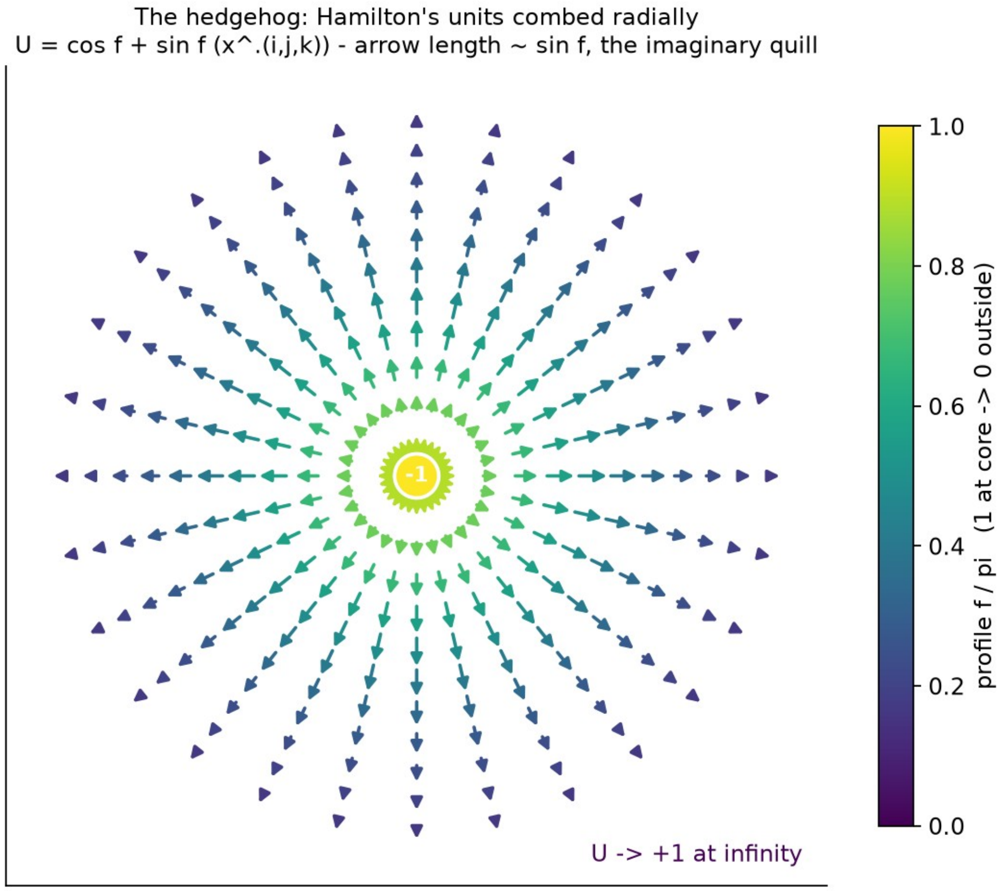

Figure 1. The hedgehog, drawn from the solved profile. Arrows show the imaginary (quaternion) component of U in a plane through the core: direction radial, length proportional to sin f. At the core U = −1 and at infinity U = +1 — both pure real, quill-less; the imaginary part peaks on the shell f = π/2. The colour scale is the profile f. The traversal from −1 through the imaginary sphere to +1, in all directions at once, wraps S^3 once: B = 1.

3. The Wall: Derrick's Theorem

Step two. Give the field the obvious energy — the gradient, how fast U varies from point to point — and ask for a stable lump. The answer is a theorem, and the proof fits in two lines.

Take any finite-energy configuration and rescale its size by a factor s: U_s(x) = U(x/s). A term in the energy with two derivatives, integrated over three-dimensional space, picks up s^3 from the volume and s^{-2} from the derivatives:

$$E_2[U_s] = s \cdot E_2[U]$$

The gradient energy *falls* as the lump shrinks — monotonically, without bound, toward zero at zero size. There is no minimum at any finite size; every winding of the space quaternion, left to the gradient alone, implodes to a point. This is Derrick's theorem (G. H. Derrick, 1964), and it is not a technicality: it is the precise mathematical identity of the series' stiffness problem. The foundations paper asks why the winding locks at a finite radius; Derrick answers that with the gradient energy alone *it cannot* — some second term, scaling the opposite way with size, must exist, and the question 'why this radius?' is the question 'what is that term, and what is its coefficient?' The floor of the Floors note must be poured, and the gradient will not set on its own.

The escape routes are constrained by the same scaling logic. A term with four derivatives scales as E_4[U_s] = E_4[U]/s — it *grows* as the lump shrinks, and can catch the collapse. A term with no derivatives (a potential) scales as s^3 and can catch the spread. And a loophole hides in the theorem's fine print: it assumes the configuration is *static*. Figure 2 shows the arithmetic of the catch.

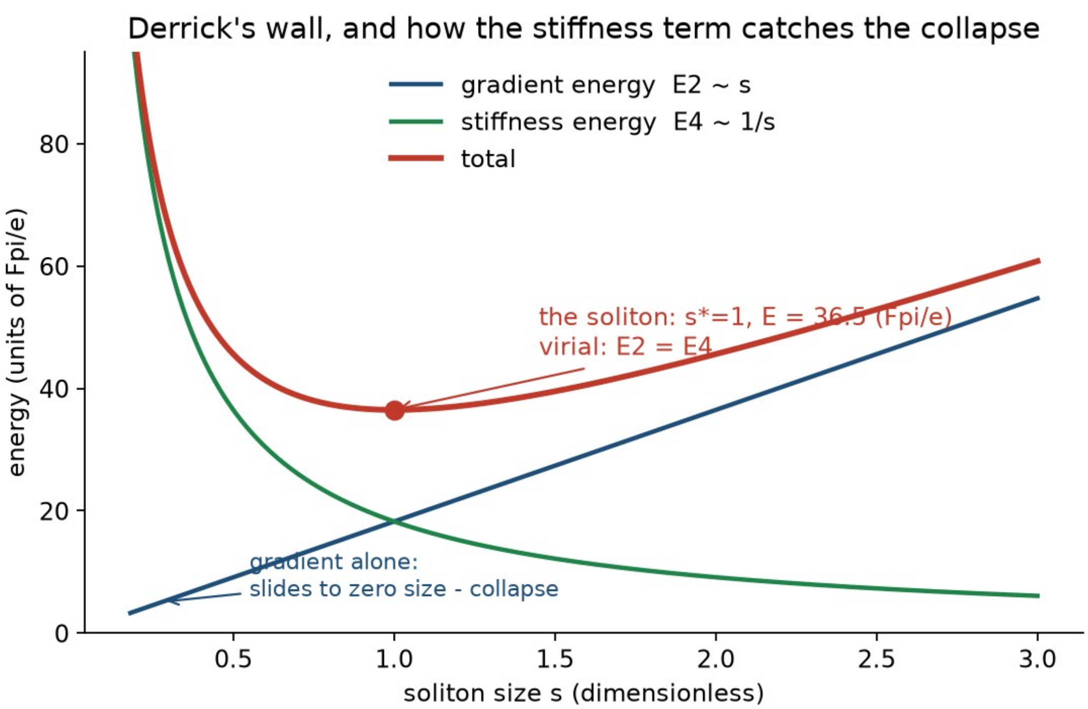

Figure 2. Derrick's wall. Against soliton size s, the gradient energy E_2 scales as s (blue): alone, it slides to zero size — collapse. A quartic stiffness term scales as 1/s (green) and catches the fall. The total (red) has its minimum where the two are equal — the Derrick virial $E_2 = E_4$ — and the computed soliton of Section 6 sits exactly there, at E = 36.5 (F_π/e).

4. The Two Doors — and the Two Classes of the Floors Note

Step three. Two escapes from Derrick are known to physics, and they map one-to-one onto the two classes of matter that the companion Floors of Condensation note distinguished for independent reasons. This convergence — the stabilization mechanisms sorting themselves into the singularity class and the trinity class — is the structural finding of this note.

Floors class	Escape from Derrick	Stabilizer	Resulting object
Singularity (lepton)	time-periodicity — the theorem assumes a static lump; the Compton clock is not static	would need a conserved charge Q and a potential fixing ω — neither exhibited	empty placeholder: $E = \hbar\omega Q$ at $\omega=\omega_C$ is circular (§4.1)
Trinity (baryon)	topology plus a quartic stiffness term	the winding number B, which no deformation can undo	a Skyrmion — computed in Section 6

4.1 The lepton door: why it is still empty

Derrick's proof silently assumes the configuration does not move. But this series' particle is never static: it winds in time at the Compton rate, $\omega_C = mc^2/\hbar$ — the tick that the Schrödinger note factors out and the Tick Count note counts. Let the internal phase rotate, $U \sim \exp(-e_3\omega t)\cdot$(profile), and the time-derivative contributes to the energy with the opposite scaling: at fixed conserved charge, rotation resists compression. Objects stabilized this way are Coleman's Q-balls (1985): non-topological lumps held together by a conserved charge and an internal rotation, whose energy in the appropriate limit is $E = \hbar\omega Q$. Apply this to the electron with its one unit of winding ($Q = 1$) turning at the Compton rate:

$$E = \hbar\, \omega_C \cdot 1 = m_e c^2 \quad \text{— but this is a tautology, not a result.}$$

This must be said bluntly, because it is the paper's weakest point and an earlier draft dressed it as a success. Since ω_C is *defined* as $m_e c^2/\hbar$, writing $E = \hbar\omega_C$ is writing $E = m_e c^2$: the mass to be explained has been inserted as the frequency, and returned unchanged. Nothing is derived. Worse, the ingredients a real Q-ball needs are simply **absent** here: Coleman's Q-balls require a specific scalar potential with the right growth and a genuine conserved global charge from which the frequency ω is determined *dynamically*; this note exhibits neither. Without a potential and a charge, there is no Q-ball — only the *name* of one, and a circular energy relation. So the honest status of the lepton door is not 'structurally identified and computationally open' but **empty**: it is a placeholder marking where a construction would go, with no construction yet in it. The one non-trivial thing that would make it real — a potential whose charge-one solution *predicts* ω_C rather than assuming it — is exactly what is missing.

4.2 The baryon door: the knot and the quartic

The second escape keeps the configuration static and adds the four-derivative term — Skyrme's move (1961). The energy becomes gradient plus stiffness; Derrick's scaling then gives $E(s) = A\cdot s + B/s$, minimized at a finite size where the two contributions are exactly equal (the virial $E_2 = E_4$ — testable in the computation, and tested). The knot cannot unwind (topology) and now cannot collapse (stiffness): a particle. This door we can walk through with a computer, and do.

5. The Construction: Energy and Equation

For the hedgehog, the Skyrme energy reduces to a functional of the single profile f. In dimensionless units $x = e \cdot F_\pi \cdot r$ — where F_π is the field's overall energy scale and e (not the electric charge) is Skyrme's dimensionless stiffness constant — the energy reads:

$$E = (F_\pi/e) \cdot 4\pi \int dx \, [\, x^2 f'^2/8 + \sin^2 f/4 \; + \; \sin^2 f \cdot f'^2 + \sin^4 f/(2x^2) \,]$$

The first bracket-pair is the gradient content E_2; the second is the stiffness content E_4. Note what the prefactor says: the whole spectrum of the model is one energy scale, F_π/e, times pure numbers — the corpus's family-law style, a scale and a geometry. Varying the functional gives the profile equation:

$$(x^2/4 + 2\sin^2 f) \cdot f'' + (x/2) \cdot f' + \sin 2f \cdot f'^2 - \sin 2f/4 - \sin^2 f \cdot \sin 2f/x^2 = 0$$

with $f(0) = \pi$, $f(\infty) = 0$, and the large-x tail $f \sim 1/x^2$. The winding number, expressed through the profile, is $B = -(2/\pi)\int \sin^2 f \cdot f' \, dx$, which evaluates to exactly 1 for any profile running from π to 0 — the topology is in the boundary conditions, not the details. One equation, one unknown function, no further input.

6. The Computation: a Particle Condenses

The profile equation is solved by shooting: integrate outward from the core with a trial slope $f'(0)$, and bisect on the slope until the trajectory neither undershoots (turning back up before reaching zero) nor overshoots (crossing below zero) — sixty bisections, a fourth-order Runge–Kutta integrator, step 0.002, and the shot lands at

$$f'(0) = -1.0048$$

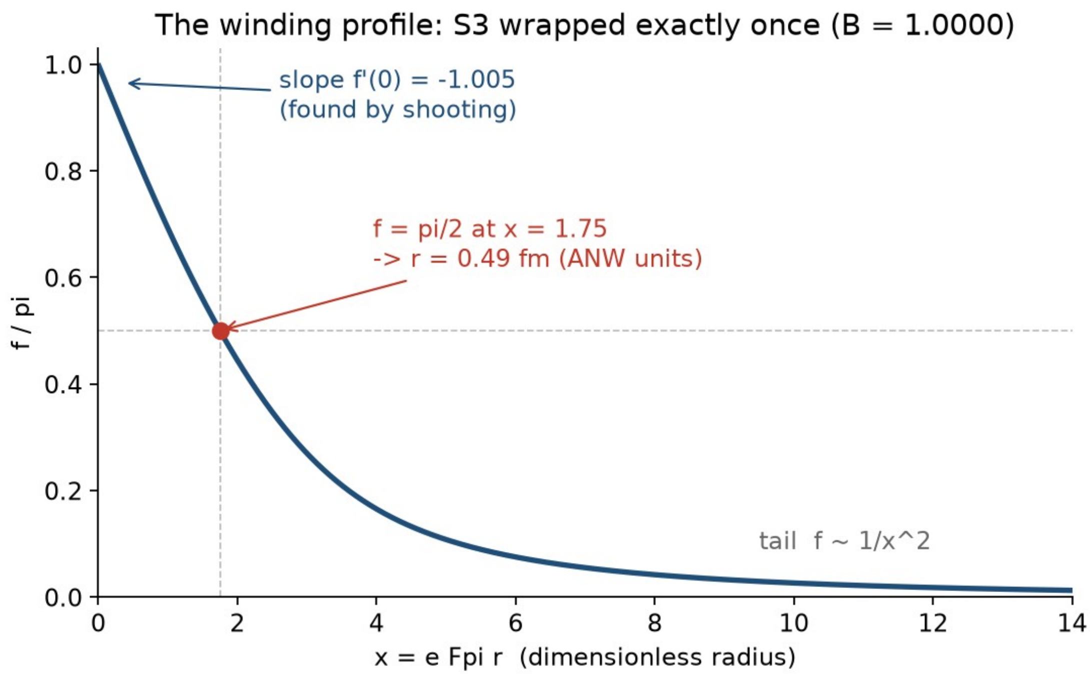

Figure 3. The solved winding profile. From full inversion at the core ($f = \pi$) the field unwinds monotonically to the identity at infinity, crossing the half-angle $f = \pi/2$ at $x = 1.75$ — which the ANW calibration places at $r \approx 0.49$ fm. The tail decays as $1/x^2$. This single curve is the entire particle.

The diagnostics of the solved configuration, each a check the construction had to pass and did:

Quantity	**Computed**	**Meaning**
winding number B	1.0000	the field wraps S^3 exactly once — baryon number, to four digits
Derrick virial E_2/E_4	0.998	gradient = stiffness at the minimum — the collapse caught in balance
energy E	36.47 (F_π/e)	the literature value (ANW 1983: 36.5) reproduced from scratch
E / topological bound $3\pi^2$	1.2319	the knot costs its topological minimum plus 23 percent
core slope f′(0)	−1.0048	the one shooting parameter
half-angle radius	x = 1.75	the particle's edge, in stiffness units

The energy stands 23 percent above the absolute floor that topology alone would permit (the Faddeev bound) — the knot is not tight, but it is close, and the excess is the price of living in three dimensions with this stiffness. Figure 4 dissects where the mass resides: the stiffness energy peaks inside the half-angle shell, the gradient energy carries the skirt, and the two integrate to *equal areas* — the Derrick balance, no longer an argument about scaling but a computed fact about a solved profile.

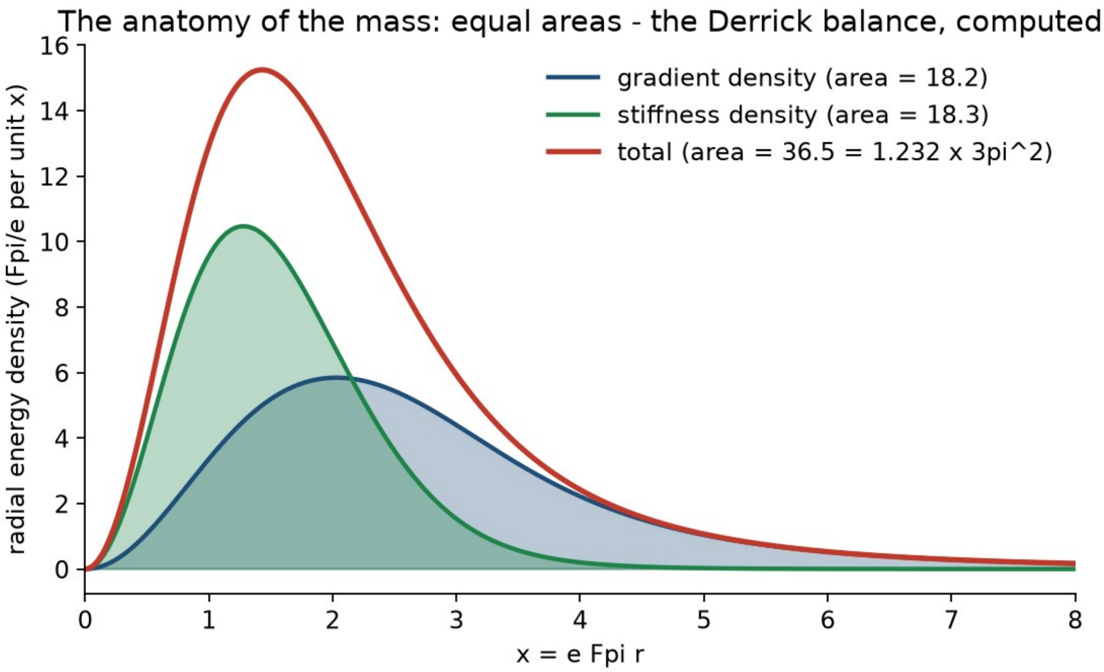

Figure 4. The anatomy of the mass. Radial energy densities of the solved soliton: gradient (blue), stiffness (green), total (red). The two shaded areas are equal to 0.2 percent — the Derrick virial, computed. The total area is the particle's mass, 36.5 in units of F_π/e.

7. Calibration, and the Spin from the Double Cover

Two constants calibrate the model to nature. Adkins, Nauenberg and Witten (1983) fixed F_π = 129 MeV and e = 5.45 by fitting the nucleon sector. With those values tonight's solution weighs

$$M = 36.47 \times F\_\pi/e = 863\ \text{MeV},$$

the classical core of the nucleon, with the half-angle radius at 0.49 fm — half a femtometre, the right room. The remaining structure is rotational. To distinguish the nucleon from the Δ(1232), ANW spin the soliton and quantize the rotation: the N and the Δ are the lowest rotational levels of the knot, split by an energy that fits at the constants above. The series has a name for this: *the mass is the dance* — the Regge logic of the companion conversation, here operating at the level of the ground state itself.

And the deepest check: spin one-half. A knot of a bosonic field has no obvious right to be a fermion — unless the topology of its configuration space grants one. It does: Finkelstein and Rubinstein (1968) showed that precisely because the B = 1 sector's loop of 2π rotation is non-contractible, the quantum theory may consistently assign the rotated state a minus sign — the knot may be quantized as spin-½, with the 4π periodicity of the double cover. The reference is not new to this series: the Pauli-exclusion paper cites the same Finkelstein–Rubinstein construction for exchange-as-rotation. The double cover that the corpus placed at the foundation of fermions is exactly the license by which this computed knot becomes one. The circle closes: the winding is the mass, the rotation is the spectrum, and the double cover is the statistics.

8. The Ledger: Priority, Duality, and the Located Debt

Priority, stated loudly. The model is Skyrme's (1960, 1961). The hedgehog solution and calibration are Adkins, Nauenberg and Witten's (1983). The legitimacy of baryon-as-soliton is Witten's large-N argument (1979, 1983): in the limit of many colours, the baryon *is* a soliton of the meson field — the quark description and the knot description are dual bookkeepings of one object. Nothing in the numerics of Sections 5–7 is original; the computation was re-performed from scratch as this series requires, and it reproduced the literature to its stated digits. What this note claims is the *reading*: that Skyrme's SU(2)-valued field is the space-quaternion phase field this framework independently arrived at; that his hedgehog is the corpus's hedgehog; that his winding number is the trinity's topological floor; and that his one fitted constant is this framework's entire remaining debt, now located.

The located debt. Skyrme's quartic term was postulated to beat Derrick, and its coefficient e = 5.45 is fitted, not derived. Every open problem of this series that touches mass — the stiffness of the weak fiber, the Higgs sector, the gearbox, why-this-radius — appears in this construction as that single number. One dimensionless constant in one term: the wall, surveyed and staked. Derrick's theorem guarantees *some* such term exists; nature has evidently chosen one; deriving its form and coefficient from the algebra is the last theorem, and it is not delivered here.

The duality owed. The corpus's proton is a trinity — three quark octonions wedged at the closure angle of the Neutron Decay paper. Tonight's proton is one knot of a quaternion field with B = 1. Large-N duality says these are two projections of one object, but the explicit map — how the tetrahedral wedge and the S^3 winding describe the same interior — is not written, and joins the ledger beside the ledger-versus-dynamical conventions of the state quaternion. Same seam, one level deeper.

Accuracy flags. The classical Skyrmion overbinds and misses at the 20–30 percent level; the pion mass term is omitted (the chiral limit); quantum corrections shift the calibration. The 863 MeV is a classical core, not a precision mass. The lepton door is identified, not computed. All of it is stated so the referee reads it here first.

9. The Program Forward

Three theorems are now cleanly owed, in ascending difficulty. (i) *The lepton computation*: exhibit the potential and the conserved charge, and solve for the profile whose charge-one solution *predicts* $\omega = \omega_C$ — the electron — rather than assuming it. The $E = \hbar\omega Q$ relation is *not* evidence here: at $\omega = \omega_C$ it is the tautology $E = mc^2$ (§4.1), so it guarantees nothing. The entire content is whether any such potential and solution exist at all; at present the lepton door is empty. (ii) *The wedge–winding map*: write the explicit correspondence between the three-octonion closure-angle description and the B = 1 quaternion knot — the two trinities made one. (iii) *The stiffness derivation*: obtain the quartic term and its coefficient from the algebra — from the two-fiber tension, the engine on the seals, or the octonion cross-coupling — rather than from a fit. The third is the gearbox; whoever pays it prices every mass in the ladder, and the deviations-from-rung that the mass-spectrum null result exposed become predictions.

10. The Sentence

A particle is a knot the space quaternion cannot untie, held at a size the stiffness will not let it flee: tonight one such knot was tied in a calculation — combed radially like the cosmos, wrapped once, balanced at the virial, weighing 863 MeV and measuring half a femtometre — and it was a nucleon; the wall between this framework and doing the same for the electron has a name, Derrick, a location, one fitted constant, and a door already ajar: the clock.

References

T. H. R. Skyrme, Proc. Roy. Soc. A 260, 127 (1961) and Nucl. Phys. 31, 556 (1962); G. H. Derrick, J. Math. Phys. 5, 1252 (1964); D. Finkelstein and J. Rubinstein, J. Math. Phys. 9, 1762 (1968); E. Witten, Nucl. Phys. B 160, 57 (1979) and Nucl. Phys. B 223, 433 (1983); G. S. Adkins, C. R. Nauenberg and E. Witten, Nucl. Phys. B 228, 552 (1983); S. Coleman, Nucl. Phys. B 262, 263 (1985) — Q-balls; L. D. Faddeev (the topological bound); and the papers and notes of this series (the Postulates; the field-equations note — the Lanczos–Conway road; the Pauli paper — Finkelstein–Rubinstein and the double cover; the Tension Medium papers — the hedgehog; the Tick Count; The Delay That Makes G; The Floors of Condensation — the two classes; the Neutron Decay paper — the closure angle whose duality with the winding is owed). Verification script: skyrmion_check.py; figures computed from the solution by skyrmion_figs.py. (Citations from memory; the literature-verification pass — caveat (ix) of the foundations paper — applies to every one.)

Acknowledgment: the numerical solution, figures, and drafting are machine work (Claude, Anthropic), performed live in conversation; the program, its readings, and its flags are the author's. The model computed here is Skyrme's; the honor of the construction belongs to Skyrme, Adkins, Nauenberg, Witten, Finkelstein and Rubinstein, whose results this note re-derived and re-read.