The State Quaternion

How a single algebraic object encodes everything an electron or proton carries

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Introduction

A particle in quantum mechanics is described by a wave function — a complex-valued function of position and time that encodes the probability of finding the particle in any given state. Wave functions live in an abstract infinite-dimensional space called Hilbert space. They are powerful but opaque. You cannot look at a wave function and immediately read off the particle's charge, its spin, its angular momentum.

We propose a different encoding. Every particle carries a state quaternion — a single four-component algebraic object in which each component corresponds to one conserved physical quantity. The state quaternion does not replace the wave function for computing interference patterns and tunnelling probabilities. But it encodes the quantum numbers — the labels that identify which state the particle is in — in a form that makes conservation laws, exclusion, and particle interactions algebraically transparent.

The structure of the state quaternion is not arbitrary. It follows directly from the octonion assignment established in Paper 5 of this series (Neutron Decay as Octonion Algebra), where each quark was assigned an octonion with eight components split into two orthogonal quaternions. The first quaternion Q_1 carries the electroweak quantum numbers: mass-energy, electric charge, spin, and angular momentum. The second quaternion Q_2 carries the colour quantum numbers: confinement, red, green, blue. For the electron, which has no colour, Q_2 is empty. The electron lives entirely in Q_1.

1. The Four Axes and What They Carry

The state quaternion has one real axis and three imaginary axes. We label them e_0 (real), e_1, e_2, e_3 (imaginary). Each axis carries one conserved quantity.

$$Q = (E/\Lambda)\cdot e_0 + q\cdot e_1 + m_s\cdot e_2 + (\ell + i\cdot m\ell)\cdot e_3 \qquad (1)$$

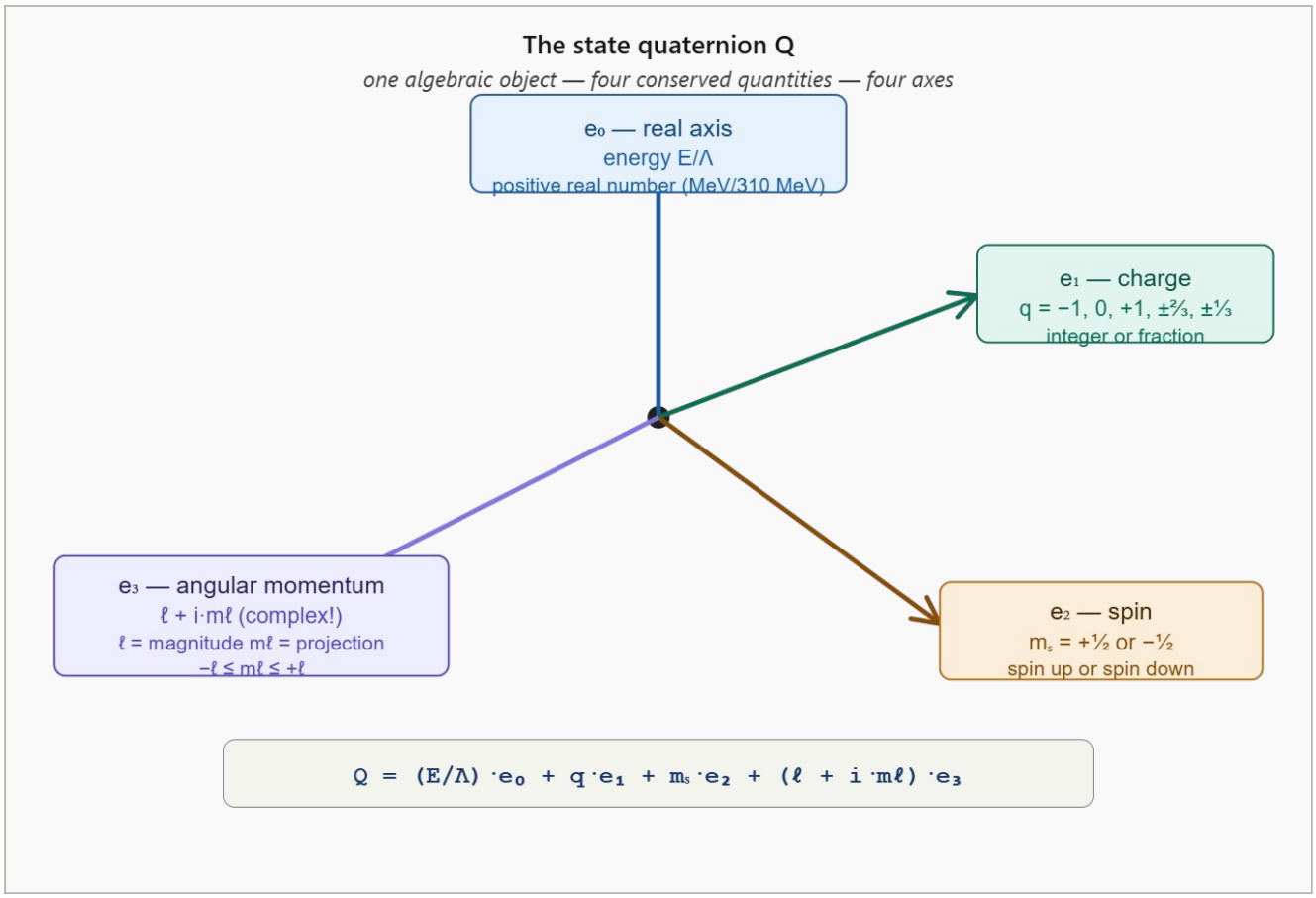


Figure 1. The state quaternion Q shown as four axes. The real axis e_0 (blue, pointing up) carries energy E/Λ. The three imaginary axes carry charge on e_1 (green), spin on e_2 (amber), and the complex angular momentum $\ell + im\ell$ on e_3 (purple). The full state quaternion is written at the bottom.

1.1 e_0 — mass-energy (real axis)

The real axis carries E/Λ, where E is the particle's rest energy in MeV and $\Lambda = 310$ MeV is the confinement scale established in Paper 5. For the electron, $E = m^e c^2 = 0.511$ MeV, giving $e_0 = 0.511/310 = 0.001648$. For the proton, $E = 938.27$ MeV, giving $e_0 = 3.0267$. The real axis carries a positive real number — a magnitude, always observable.

1.2 e_1 — electric charge

The first imaginary axis carries the electric charge in units of the elementary charge e. For the electron $q = -1$. For the proton $q = +1$. For a neutron $q = 0$. For an up quark $q = +2/3$. For a down quark $q = -1/3$. Charge is a discrete quantum number — an integer or simple fraction — fixed for each particle species. It does not take different values in different quantum states of the same particle.

1.3 e_2 — spin projection

The second imaginary axis carries m_s, the spin projection along the quantisation axis. For a spin-½ particle (electron, proton, quark), $m_s = +½$ or $m_s = -½$. Spin is the intrinsic angular momentum of the particle — not associated with orbital motion, but an internal degree of freedom. The two values +½ and −½ are the only options for all the particles we consider here.

1.4 e_3 — orbital angular momentum (complex)

The third imaginary axis carries a complex number. This is the key structural insight of this paper.

$$L = \ell + i \cdot m\ell \qquad \ell = 0, 1, 2, \ldots \qquad -\ell \leq m\ell \leq +\ell \qquad (2)$$

The real part ℓ is the orbital angular momentum quantum number — the total magnitude of the orbital angular momentum, independent of which direction you choose as your quantisation axis. It takes non-negative integer values: 0, 1, 2, 3, ... In atomic physics these are labelled s, p, d, f for $\ell = 0, 1, 2, 3$.

The imaginary part $m\ell$ is the projection of the orbital angular momentum along the quantisation axis. It is coordinate-dependent: if you rotate your frame, $m\ell$ changes. It takes integer values from $-\ell$ to $+\ell$, giving $2\ell + 1$ possible values for each ℓ.

This is exactly the structure of a complex number: the real part is the invariant magnitude, the imaginary part is the projection. The modulus $|\ell + im\ell| = \sqrt{(\ell^2 + m\ell^2)}$ combines both. This is also the same structure as the spacetime biquaternion established in Paper 1, where the time component $W = i\tau$ carries a complex number on the real axis: an invariant on the real part, a coordinate-dependent projection on the imaginary part.

1.5 Why charge and spin are not complex

Charge (e_1) does not have a magnitude and a projection. It is a topological quantum number that does not change when you rotate the coordinate system. There is no direction to project charge along. It is the same from every angle. So e_1 carries a real integer or fraction.

Spin (e_2) does have a projection, but for a spin-½ particle the magnitude is fixed at ½ always — it never changes between states. Only the sign of the projection varies: +½ or −½. So e_2 carries a real number with two possible values, not a complex number.

Angular momentum (e_3) has both a variable magnitude ℓ and a variable projection $m\ell$, both changing between quantum states. Only e_3 needs two real degrees of freedom, and a complex number is the natural way to carry them on a single axis.

The assignment — why each axis carries what it carries

e_0 (real): energy E/Λ — a real positive magnitude. The observable.
e_1 (imaginary): charge q — an integer. No direction to project. Always the same.
e_2 (imaginary): spin m_s — fixed magnitude ½, only sign varies. Two values only.
e_3 (imaginary): angular momentum $\ell + im\ell$ — both magnitude and projection vary.
Complex number on one axis: two degrees of freedom, one slot.

2. The Electron State Quaternion — Explicit States

The hydrogen atom has energy levels labelled by the principal quantum number n = 1, 2, 3, ... The energy of level n is:

$$E_n = -13.606\ \text{eV} / n^2 \qquad n = 1, 2, 3, \ldots \qquad (3)$$

The e_0 component of the state quaternion uses the electron's total energy — rest energy plus orbital energy — divided by Λ. The orbital correction $-13.606/n^2$ eV is only parts per million of the rest energy 0.511 MeV, and at display precision every hydrogen state prints as $e_0 = 0.511/310 = 0.001648$. But the correction must not be truncated from the state itself: $e_0(n) = (m_e c^2 - 13.606\ \text{eV}/n^2)/\Lambda$. The 26.6 ppm shift is precisely the information that distinguishes the shells. Without it, 1s↑ and 2s↑ would carry literally identical quaternions on all four axes, and the exclusion criterion of Chapter 3 would forbid lithium — a third electron could never enter 2s while 1s is filled. With e_0 carrying the level term, no two states of different n are identical, and exclusion operates between shells exactly as it does within them. An apparent approximation becomes a point of principle: no component of the state quaternion may be truncated, because identity of states is exactly what the framework is about.

2.1 The ground state

The ground state of hydrogen has n=1, ℓ=0, $m\ell$=0, m_s=+½:

$$Q(1s,\uparrow) = 0.001648 \cdot e_0 - e_1 + \tfrac{1}{2} \cdot e_2 + 0 \cdot e_3 \qquad (4)$$

The e_3 component is zero because both $\ell = 0$ and $m\ell = 0$. The electron has no orbital angular momentum. Its orbital is spherically symmetric — the 1s orbital.

A second electron can occupy the same spatial orbital with opposite spin $m_s = -\tfrac{1}{2}$. Its quaternion differs only on e_2. These two electrons, one spin-up and one spin-down, fill the n=1 shell. A third electron cannot enter: it would have to be identical to one of these two (same e_0, e_1, e_2, e_3 on all axes), and $Q \wedge Q = 0$ forbids it.

2.2 The n=2 shell — eight distinct states

The n=2 shell contains four distinct orbital states before spin is included: the 2s orbital (ℓ=0, mℓ=0) and three 2p orbitals (ℓ=1, mℓ=−1,0,+1). Each combines with two spin states, giving eight states total. Their state quaternions:

$$Q(2s) = 0.001648 \cdot e_0 - e_1 + m_s \cdot e_2 + (0 + 0i) \cdot e_3 \quad (5)$$

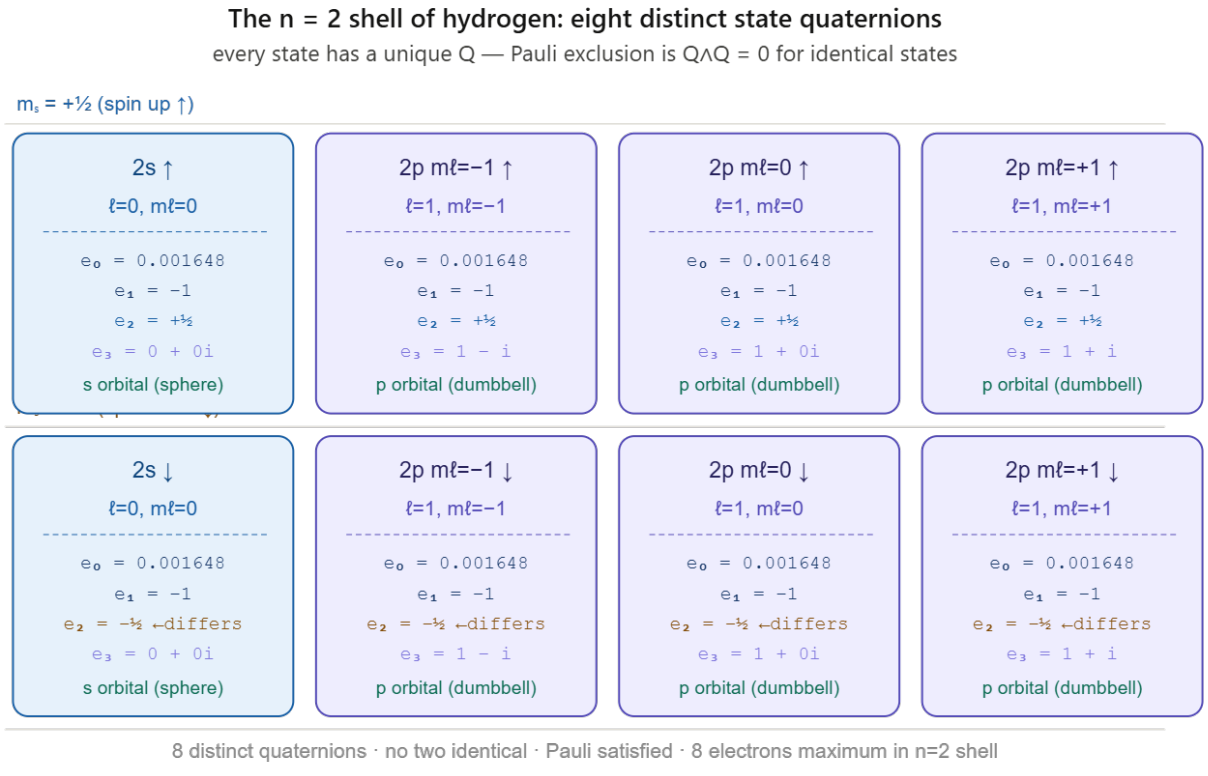
$$Q(2p, m\ell{=}-1) = 0.001648 \cdot e_0 - e_1 + m_s \cdot e_2 + (1 - i) \cdot e_3 \quad (8)$$

$$Q(2p, m\ell{=}0) = 0.001648 \cdot e_0 - e_1 + m_s \cdot e_2 + (1 + 0i) \cdot e_3 \quad (6)$$

$$Q(2p, m\ell{=}+1) = 0.001648 \cdot e_0 - e_1 + m_s \cdot e_2 + (1 + i) \cdot e_3 \quad (7)$$

Each of these four orbital states combines with $m_s = +½$ and $m_s = -½$, giving eight quaternions in total. Every one is distinct — no two are identical on all four axes. The Pauli exclusion principle is satisfied by inspection: any attempt to put a ninth electron into this shell would require it to duplicate one of the eight, giving zero wedge product and zero amplitude.

Note what the complex e_3 component achieves. The 2s state has $e_3 = 0$. The three 2p states all have real part $\ell = 1$. They are distinguished by their imaginary parts $m\ell = -1, 0, +1$. Without the complex structure, the three 2p states (all with $\ell = 1$) would be indistinguishable on e_3, and two of the eight states would be identical. Pauli would break. The complex angular momentum on e_3 is not a formal convenience — it is necessary for Pauli exclusion to work.



[Figure 2 — Eight n=2 hydrogen states as state quaternions]

Figure 2. The eight state quaternions of the n=2 hydrogen shell. Top row (spin up): 2s, 2p mℓ=−1, 2p mℓ=0, 2p mℓ=+1. Bottom row (spin down): same four orbitals with e_2 flipped to −½. Each box shows all four components. No two boxes are identical. Pauli exclusion is satisfied for all 28 pairs.

State	e_0	e_1	e_2	$e_3 = \ell + im\ell$	$\lvert e_3 \rvert$
2s ↑	0.001648	−1	+½	0 + 0i	0
2s ↓	0.001648	−1	−½	0 + 0i	0
2p mℓ=−1 ↑	0.001648	−1	+½	1 − i	√2
2p mℓ=−1 ↓	0.001648	−1	−½	1 − i	√2
2p mℓ=0 ↑	0.001648	−1	+½	1 + 0i	1
2p mℓ=0 ↓	0.001648	−1	−½	1 + 0i	1
2p mℓ=+1 ↑	0.001648	−1	+½	1 + i	√2
2p mℓ=+1 ↓	0.001648	−1	−½	1 + i	√2

3. The Pauli Exclusion Principle as Q∧Q = 0

The wedge product (antisymmetric product) of two quaternions Q and P is:

$$Q \wedge P = \tfrac{1}{2}(QP - PQ); \quad \text{for identical states } P = Q: \quad Q \wedge Q = 0 \tag{9}$$

Setting P = Q gives Q ∧ Q = ½(QQ − QQ) = 0 by antisymmetry. The identity itself is a theorem of quaternion algebra — though, taken alone, it is a theorem about any element of any algebra. Two particles cannot occupy the same state because the antisymmetric product of identical state quaternions is zero — the two-particle state has zero amplitude.

The physical postulate must therefore be stated plainly, in the same spirit as the W = iτ relabeling of Paper 1: what carries the physics is the rule that the two-particle amplitude is the wedge — antisymmetrized — product of the two state quaternions. Q ∧ Q = 0 alone is empty; combined with that postulate and with the encoding of Sections 1–2, distinctness of quantum numbers becomes distinctness of quaternions, and exclusion becomes mechanical. The postulate is relocated into a natural algebraic operation, not eliminated.

3.1 Verification for helium

Helium has two electrons in the ground state. Their state quaternions:

Q↑ = [0.001648, −1, +½, 0] (1s spin-up)

Q↓ = [0.001648, −1, −½, 0] (1s spin-down)

These differ on e_2: +½ versus −½. They are not identical. Therefore $Q\uparrow \wedge Q\downarrow \neq 0$, and two electrons coexist in the ground state. ✓

A third electron attempting to enter the ground state would have either $m_s = +½$ (identical to Q↑, giving $Q \wedge Q\uparrow = 0$, forbidden) or $m_s = -½$ (identical to Q↓, giving $Q \wedge Q\downarrow = 0$, forbidden). The third electron must go to n=2. This is the shell structure of the periodic table, emerging from $Q \wedge Q = 0$.

3.2 The 2p orbital: why complex L matters for Pauli

Without complex angular momentum, the three 2p states (ℓ=1, $m\ell$=−1,0,+1) all have $e_3 = 1$ (real part only). The spin-up 2p state would then be:

Q(2p,↑) = [0.001648, −1, +½, 1] (same for all three mℓ values — indistinguishable!)

Three different physical states — $m\ell = -1, 0, +1$ — would map to the same quaternion. Pauli would be violated: three electrons with the same quaternion would all have $Q \wedge Q = 0$. Only by making e_3 complex — carrying both ℓ and $m\ell$ — do the three states become distinct and Pauli remain intact.

Pauli exclusion from Q∧Q = 0 — requires complex e_3
Without complex e_3: 2p mℓ=−1, 0, +1 all give $e_3 = 1$ → identical quaternions → Pauli violated
With complex e_3: 1−i, 1+0i, 1+i → three distinct quaternions → Pauli satisfied
The complex structure on e_3 is forced by the exclusion principle, not assumed.

Two caveats complete the statement of the rule. First, the wedge product vanishes not only for identical quaternions: $Q \wedge P = 0$ whenever the imaginary parts of Q and P are parallel as 3-vectors. Within a given shell the encodings above are safe — the shared e_1 component pins the scale, so parallel forces identical — but the criterion “distinct implies allowed” requires this parallel-case check, stated here explicitly. Second, the state quaternion carries no position or orbital-overlap information. As written, the rule would forbid two ground-state hydrogen atoms anywhere in the universe from sharing quantum numbers. The standard scope condition — the two particles must occupy the same spatial orbital, with overlapping wavefunctions — has to be supplied from outside; it is information Q does not carry. That is a limitation of the encoding, acknowledged rather than hidden.

4. Selection Rules from Conservation on Each Axis

Every atomic transition — every photon emitted or absorbed — must preserve the state quaternion’s components subject to what the photon carries away. The photon’s state quaternion is:

$$Q\gamma = (h\nu/\Lambda)\cdot e_0 + 0\cdot e_1 + 0\cdot e_2 + (1 + i\cdot m)\cdot e_3 \qquad m = -1, 0, +1 \tag{10}$$

The photon carries energy (e_0), no charge ($e_1 = 0$), and one unit of angular momentum — its spin — with projection $m = -1, 0, +1$, entered once, on the angular-momentum axis e_3. Entering the photon's spin a second time as $e_2 = \pm 1$ would double-count it; worse, an additively conserved $e_2 = \pm 1$ would force $\Delta m_s = \mp 1$, which is impossible for a spin-½ electron. Conservation at each axis gives the selection rules:

$$\Delta e_0: \; E_{initial} = E_{final} + h\nu \qquad \text{(energy conservation)} \tag{11}$$

$$\Delta e_1: \; q_{initial} = q_{final} \qquad \text{(charge conservation)} \tag{12}$$

$$\Delta e_2: \; \Delta m_s = 0 \qquad \text{(the dipole operator does not act on the spin axis)} \tag{13}$$

$$\Delta e_3: \; \Delta \ell = \pm 1 \quad \text{and} \quad \Delta m_\ell = 0, \pm 1 \qquad \text{(angular momentum)} \tag{14}$$

The selection rule $\Delta\ell = \pm 1$ — the most important rule in atomic spectroscopy, the one that determines which transitions are electric dipole allowed — falls directly from equation (

(14) the e_3 conservation equation. The photon carries one unit of orbital angular momentum (real part of $e_3 = 1$), so the electron's ℓ must change by exactly ± 1. The e_3 axis makes the rule transparent — it lives on a single axis. Two limits must be stated honestly. First, componentwise complex addition is not the full machinery by which angular momenta couple: the correct rule is the quantum triangle rule together with the parity of the dipole operator, and it is parity — a quantum number the state quaternion does not carry — that actually forbids $\Delta\ell = 0$. Second, the additive bookkeeping is exact only in the leading cases: for $3d(m_\ell = +2) \rightarrow 2p(m_\ell = +1)$, componentwise subtraction would require a photon e_3 of modulus ≈ 1.08 rather than $\sqrt{2}$. The axis structure organizes the selection rules; the addition law behind them comes from the underlying operator algebra.

The rule $\Delta m_\ell = 0, \pm 1$ comes from the imaginary part of e_3: the photon's imaginary part is $m = -1, 0$, or $+1$, so the electron's m_ℓ changes by at most ± 1.

Spectroscopic selection rules from axis conservation
$\Delta\ell = \pm 1$: from conservation of the real part of e_3 (photon carries $\ell\gamma = 1$)
$\Delta m_\ell = 0, \pm 1$: from conservation of the imaginary part of e_3 (photon carries $m = -1, 0, +1$)
$\Delta m_s = 0$: the electric-dipole operator does not act on the spin axis e_2
$\Delta q = 0$: from conservation of e_1 (photon has no charge)
Each rule lives on one axis: the four-axis structure organizes the bookkeeping. The addition law and the parity input behind the rules come from the underlying operator algebra.

5. The Quaternion Norm

The squared norm of the state quaternion is:

$$|Q|^2 = (E/\Lambda)^2 + q^2 + m_s^2 + \ell^2 + m\ell^2 \qquad (15)$$

For the hydrogen ground state (n=1, ℓ=0, mℓ=0, m_s=+½):

$$|Q(1s,\uparrow)|^2 = (0.001648)^2 + 1 + \tfrac{1}{4} + 0 \approx 5/4$$

For the n=2, ℓ=1, mℓ=+1, spin-up state:

$$|Q(2p,m\ell{=}{+}1,\uparrow)|^2 = (0.001648)^2 + 1 + \tfrac{1}{4} + (1^2{+}1^2) \approx 13/4$$

The norm grows with excitation. When a photon is emitted, the components of the electron's state quaternion change by what the photon carries away, axis by axis — the bookkeeping of Chapter 4. The norms themselves do not subtract: $|Q|^2$ is quadratic, and norms are not additive. For 2p → 1s the electron's squared norm drops by 13/4 − 5/4 = 2, while the photon's squared norm from equations (10) and (15) is ≈ 3. The balance is componentwise, not norm-wise. Energy, angular momentum, and spin are transferred from particle to particle, and each axis adjusts accordingly. The norm is the measure of how much quantum state the particle is carrying.

6. The Proton State Quaternion — at the Surface

Before an electron probe penetrates the proton's interior, the proton appears as a point particle with definite quantum numbers. At the surface, it has a state quaternion with the same structure as the electron's:

$$Q_p = 3.0267\cdot e_0 + (+1)\cdot e_1 + m_s\cdot e_2 + (0+0i)\cdot e_3 \quad \text{[surface]} \qquad (16)$$

The proton is 1836.1 times heavier than the electron. This ratio appears directly as the ratio of their e_0 components: 3.0267/0.001648 = 1836.1. The mass ratio is transparent in the state quaternion.

6.1 The electron-proton interaction at the surface

The Coulomb potential energy between electron and proton is given by the product of their e_1 components:

$$U(r) = (q^e\cdot q_p\cdot\alpha\cdot\mathcal{H}c)\,/\,r = (-1)(+1)\times 1.4400\ \text{MeV}\cdot\text{fm}\,/\,r \qquad (17)$$

The product of e_1 components is $q^e \times q_p = (-1)(+1) = -1$. A negative product means attraction. Two protons give (+1)(+1) = +1: repulsion. The sign

of the Coulomb interaction is the product of the e_1 components. The magnitude is $\alpha\hbar c/r \approx 1.44/r$ MeV·fm.

At the Bohr radius a_0 = 52,918 fm, $U(a_0) = -1.44/52918 = -27.2$ eV. Half of this (the virial theorem contribution from the electron's kinetic energy) gives the binding energy 13.6 eV. The entire hydrogen spectrum follows from the e_1 components and the quantisation condition on e_0 from equation (

(3).

7. The Boundary: Where the Quaternion Hands Over to the Octonion

The state quaternion Q_1 gives a complete description of the electron and of the proton's surface as long as the electron's de Broglie wavelength is large compared to the proton radius r_p = 0.8414 fm. The transition occurs at:

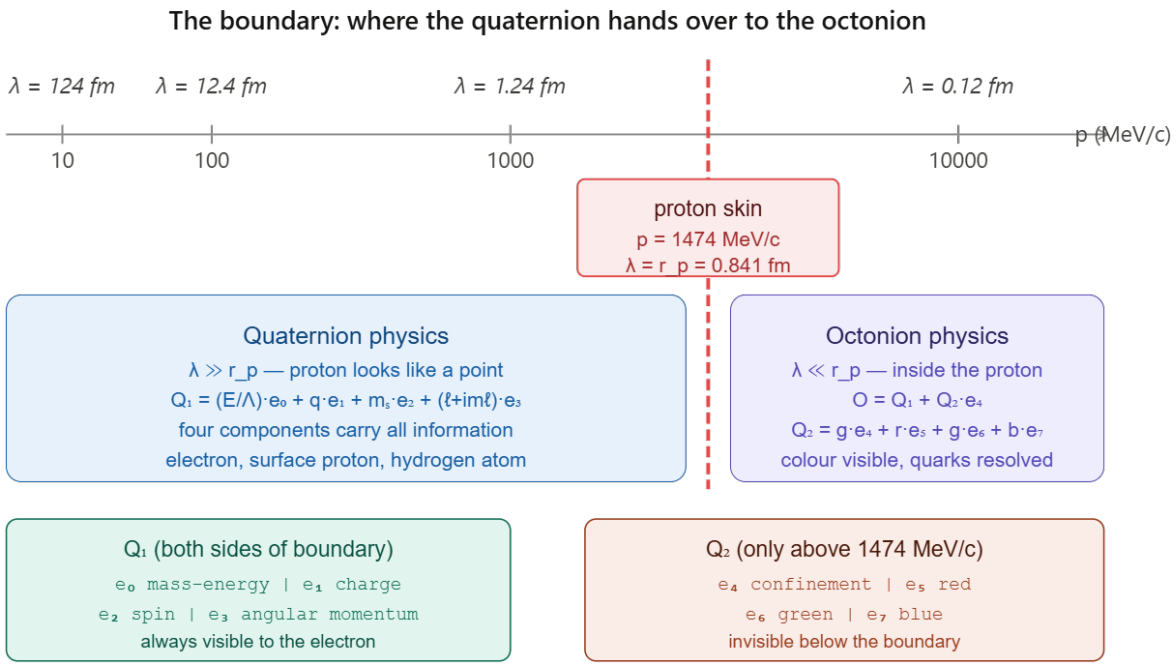
$$p\_transition = 2\pi\hbar c / r_p = 2\pi \times 197.327 / 0.8414 = 1474 \text{ MeV/c} \quad (18)$$

Below this momentum, Q_1 is all that matters. The proton is a point with a charge, a spin, and a mass — a state quaternion exactly like equation (

(16). Above this momentum, the electron's wavelength shrinks below the proton radius. It begins to see inside. The proton's Q_2 — the colour quaternion, the second orthogonal quaternion of the full octonion — becomes visible. The four components of Q_1 are no longer sufficient. You need all eight components of the octonion:

$$O_p = Q_1 + Q_2 \cdot e_4 \qquad Q_2 = g\cdot e_4 + r\cdot e_5 + g\cdot e_6 + b\cdot e_7 \qquad (19)$$

where Q_2 carries the confinement strength and the colour charges of the three constituent quarks. The full component assignment for the proton's octonion is given in Paper 5 (Section 2). The six quark presets, the closure angle, the spring constant, and the neutron-proton mass difference all emerge from the structure of Q_2.



[Figure 3 — Quaternion to octonion boundary at p = 1474 MeV/c]

Figure 3. The boundary between quaternion and octonion physics on a logarithmic momentum axis. Left of the red dashed line ($p < 1474$ MeV/c, $\lambda > r_p$): the proton looks like a point and Q_1 carries all information. Right of the boundary ($p > 1474$ MeV/c, $\lambda < r_p$): the colour quaternion Q_2 becomes visible. The electron, which has no colour, lives entirely in Q_1 at all energies.

The state quaternion — complete summary
$Q = (E/\Lambda)\cdot e_0 + q\cdot e_1 + m_s\cdot e_2 + (\ell + im\ell)\cdot e_3$

e_0: energy (real, positive, in units of Λ = 310 MeV)
e_1: charge (integer or simple fraction, discrete)
e_2: spin projection (+½ or −½ for spin-½ particles)
e_3: orbital angular momentum ($\ell + im\ell$, complex)
real part ℓ: total magnitude, invariant under rotation
imaginary part $m\ell$: projection, coordinate-dependent

Pauli exclusion: two-particle amplitude = wedge product (postulate); $Q \wedge Q = 0$ (theorem)
Selection rules: bookkeeping on the four axes when a photon is emitted (Chapter 4)
Proton surface: same structure, $e_0 = 3.0267$, $e_1 = +1$
Transition at p = 1474 MeV/c: $Q_1 + Q_2\cdot e_4$ — the octonion

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Equation Index

(1) $Q = (E/\Lambda)\cdot e_0 + q\cdot e_1 + m_s\cdot e_2 + (\ell + i\cdot m\ell)\cdot e_3$

(2) $L = \ell + i{\cdot}m\ell \qquad \ell = 0, 1, 2, \ldots \qquad -\ell \le m\ell \le +\ell$

(3) $E_n = -13.606\ \text{eV} / n^2 \qquad n = 1, 2, 3, \ldots$

(4) $Q(1s,\uparrow) = 0.001648{\cdot}e_0 - e_1 + \tfrac{1}{2}{\cdot}e_2 + 0{\cdot}e_3$

(5) $Q(2s) = 0.001648{\cdot}e_0 - e_1 + m_s{\cdot}e_2 + (0 + 0i){\cdot}e_3$

(6) $Q(2p, m\ell{=}0) = 0.001648{\cdot}e_0 - e_1 + m_s{\cdot}e_2 + (1 + 0i){\cdot}e_3$

(7) $Q(2p, m\ell{=}{+}1) = 0.001648{\cdot}e_0 - e_1 + m_s{\cdot}e_2 + (1 + i){\cdot}e_3$

(8) $Q(2p, m\ell{=}{-}1) = 0.001648{\cdot}e_0 - e_1 + m_s{\cdot}e_2 + (1 - i){\cdot}e_3$

(9) $Q \wedge P = \tfrac{1}{2}(QP - PQ)$; for identical states $P = Q$: $Q \wedge Q = 0$

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