# The Stiffness Is the Curvature
# Die Steifigkeit ist die Krümmung

The Skyrme Term as the Curvature of the Field's Target Sphere, and the Coefficient as the Door to the Next Mode

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## Abstract

The particle paper of this series ended at one fitted constant: Skyrme's stiffness e = 5.45, the term that catches Derrick's collapse and sets the size of every knot, postulated there as it was postulated in 1961. This note removes the postulate and prices the constant. First, an identity of pure geometry, walked symbol by symbol: the Skyrme term is the square of the pulled-back curvature two-form of the target sphere. For a sphere of constant curvature, the curvature two-form is vielbein wedge vielbein, $R^{ab} = e^a \wedge e^b$; its pullback by the winding field is the commutator of the currents; its square is Skyrme's term, exactly. One boundary must be drawn at the outset, because it is the paper's most abusable line and an earlier draft crossed it: *this is the curvature of the field's target sphere $S^3$ — the internal manifold the winding field U maps into — computed by pullback, exactly as verified in frame_quaternion_verification.py. It is a genuine second geometric invariant of the winding, so that the two invariants of a wound field are its pulled-back target-metric (tension) and its pulled-back target-curvature (stiffness). But it is not automatically the foundation theory's spacetime frame curvature $R = d\omega + \omega \wedge \omega$ (the curvature of the gravitational connection): those are two different manifolds' curvatures, and identifying them is a further claim this paper does not make. What is established is narrower and clean: Skyrme's postulated quartic term is the squared pulled-back target curvature — a standard differential-geometry identity — so within the effective field it is not an alien addition but the natural second invariant. The debt thereby shrinks from a term to a number — the ratio of the curvature-energy price to the tension — and for the one fiber whose cellar physics can enter (the pion field of QCD), that number has a known and verifiable address: the first excited mode of the tower, the ρ meson. Hidden-local-symmetry theory generates the Skyrme term by integrating out the ρ, predicting $e \approx g_{\rho\pi\pi}$; this note verifies the prediction against data: g = 5.976 from the measured ρ width, g = 5.952 from the KSRF mass relation, against the fitted e = 5.45 — agreement at 9 percent, with the stiffness length unit $1/(eF_\pi)$ = 0.281 fm landing within 10 percent of the ρ's Compton radius 0.255 fm. The pattern this establishes — the stiffness of a rung is the door to its next mode; the amputated interior returns as a tower whose ground floor sets the coefficient — is then set beside — not extended to — the weak fiber, and only as a flagged, non-viable curiosity: the lowest computed weak tone, $(3/2)m_W$ = 120.6 GeV, lies 3.7 percent from the

*Higgs, but that tone is a Dirac (spinorial) eigenmode while the Higgs is a scalar, and a spin-½ mode cannot be a spin-0 particle. That categorical mismatch blocks the identification, so the pattern has exactly* one* demonstrated instance — the pion fiber and its ρ (a genuine scalar-isovector state) — not two; the weak comparison is a number without a mechanism until a scalar mode of the wound weak $S^3$ is exhibited. Priority throughout belongs to the literature — Skyrme; Bando, Kugo and Yamawaki; the KSRF relation — and is stated loudly; the contribution claimed is the identification of the pair, the translation to the ladder, and the relocation of the stiffness problem from 'why is there a term?' to 'why does the next mode live where it lives?' — which is the same question one rung higher, exactly as a ladder should feel. Every symbol introduced before use; every flag flown.

## 1. The Question Left Standing

The particle paper built a baryon from the space quaternion and closed on one honest embarrassment: the quartic term that stops Derrick's collapse was *postulated*, and its dimensionless coefficient e = 5.45 *fitted*. Every scale of the knot — its size, its mass, the whole calibration — hangs on that one number. Two readings were possible. Either the term is an alien addition, bolted on because Derrick demands something, in which case the framework has merely relocated its ignorance; or the term is a natural invariant the formalism already owns, in which case only the coefficient is owed. This note establishes the second reading by identity, then prices the coefficient by measurement.

## 2. The Two Objects of the Frame Note

Recall the two objects the field-equations note placed at the base of the series. The **frame quaternion** θ: at each point, the local clock and rulers — the vielbein, whose norm is the metric, $ds^2 = \theta\cdot\theta$. And the **curvature** $R = d\omega + \omega\wedge\omega$: the rotation a frame silently accumulates when carried around a small loop. The note's template runs 'curvature of the winding equals source.' Add to this Postulate P2's anatomy of a world: every world of the ladder is a sphere of *constant* curvature — 'a world does not have a curvature profile; it has a curvature.' These are the only ingredients the identity needs.

## 3. The Identity, Walked Slowly

Let U(x) be the winding field of the particle paper — at every point of space, a unit quaternion, a point on the three-sphere $S^3$. Every symbol in what follows is built from U.

### 3.1 The currents are pulled-back vielbeins

Define the currents L_μ = U†∂_μU. They are valued in the algebra (pure quaternion-imaginary), with components L^a_μ on the units: L_μ = L^a_μ T_a, where T_a are the quaternion units in their 2×2 representation and a runs over 1, 2, 3. Geometrically the L^a_μ are the *vielbeins of the target sphere, pulled back to space by the map U*: they measure how far, per unit step in direction μ, the field moves along the target's a-th axis. The gradient energy of the winding is their square:

$$E_2 = (F\_\pi^2/16) \int d^3x \;\; \Sigma\_a\, L\text{^}a\_\mu\, L\text{^}a\_\mu \;\; = \;\; |\text{pullback of } \theta|^2$$

— the pulled-back *metric* of the sphere, weighted by the tension $F\_\pi^2$. This is the first object of the frame note, worn by the winding.

### 3.2 The sphere's curvature is vielbein wedge vielbein

Now P2, taken literally. For a sphere of constant curvature (unit radius), the curvature two-form is not an independent field; it is built from the vielbeins themselves:

$$R\text{^}\{ab\} = e\text{^}a \wedge e\text{^}b$$

This single line is the geometric content of 'a sphere has one curvature everywhere': the rotation accumulated around a small loop is proportional to the loop's area, with a universal constant of proportionality — curvature equals area form, componentwise e^a∧e^b. It is the defining property of the constant-curvature worlds that P2 assigns to every rung of the ladder.

### 3.3 Pull back, and square

Pull the target's curvature two-form back to space along U — substitute the pulled-back vielbeins:

$$(U^*R\text{^}\{ab\})\_\{\mu\nu\} = L\text{^}a\_\mu\, L\text{^}b\_\nu - L\text{^}a\_\nu\, L\text{^}b\_\mu$$

And compare with the commutator of the currents. Since [T_a, T_b] = ε_{abc}T_c (Hamilton's table),

$$[L\_\mu, L\_\nu] = \varepsilon\_\{abc\}\, L\text{^}a\_\mu\, L\text{^}b\_\nu\, T\_c\,,$$

whose square, traced, is componentwise identical (up to a fixed numerical factor) to the square of the pulled-back curvature:

$$Tr(\, [L\_\mu, L\_\nu][L\text{^}\mu, L\text{^}\nu]\, ) \;\propto\; \Sigma\_\{ab\}\, (\, L\text{^}a\_\mu L\text{^}b\_\nu - L\text{^}a\_\nu L\text{^}b\_\mu\, )^2 \;=\; |U^*R|^2$$

The left side is **Skyrme's term**. The right side is **the curvature energy of the wound sphere**. They are the same object:

$$E = (\text{tension}) \cdot |\text{pullback } \theta|^2 \;+\; (\text{stiffness}) \cdot |\text{pullback } R|^2$$

**The pair (tension, stiffness) is the pair (target metric, target curvature)** of the field's $S^3$ — the pulled-back $|\theta\_\text{target}|^2$ and $|R\_\text{target}|^2$. Skyrme, in 1961, without the language, wrote down the metric energy and the curvature energy of a wound constant-curvature *target* sphere. The debt shrinks accordingly: not 'why is there a quartic term?' — the target geometry supplies exactly this one, as its second invariant — but 'what is the ratio of its coefficient to the tension?' One number. (What this does **not** do: it does not show the stiffness is the foundation theory's *spacetime* curvature R = dω + ω∧ω. The suggestive echo — a wound sphere carries a metric energy and a curvature energy in both the target and the spacetime settings — is a formal parallel, not an identity; asserting the identity would require deriving the pion field's target $S^3$ *from* the spacetime frame, which this paper does not do.)

### 3.4 The Maurer–Cartan subtlety, stated so the note does not overclaim

One piece of craftsmanship must be recorded. The *composite* connection built from the full currents is pure gauge, and its total curvature vanishes identically — the Maurer–Cartan identity $dL + L \wedge L = 0$. The nonvanishing object in Section 3.3 is the *target's* curvature pulled back — equivalently, in the symmetric-space (coset) decomposition of the sphere, the curvature of the coset part, for which $R = -[e, e]$ is exactly the constant-curvature statement used above. The distinction is standard differential geometry of symmetric spaces; it changes no result, but a referee will ask, and the answer is written here first.

## 4. What Remains: One Number

With the form fixed by geometry, dimensional analysis isolates the residue. The tension carries $F_\pi^2$ (an energy scale squared); the curvature term carries its own coefficient $1/e^2$ (dimensionless, since four derivatives need no scale). The Derrick balance of the particle paper then sets the knot's size at the geometric mean, $R \sim$ *$1/(eF_\pi)$, and its mass at $F_\pi/e$ times a pure number (computed there: 36.47). Everything is fixed except e itself: the price of a unit of fiber curvature, measured in units of the fiber's tension.** The remainder of this note is about that price.

## 5. The Number's Address: the One Cellar Physics Can Enter

For the pion field — the chiral field of QCD, of which the Skyrme model is the effective facade — the cellar is known, and the coefficient has been *derived* rather than fitted. The mechanism (hidden local symmetry; Bando, Kugo and Yamawaki) is precisely the storeys picture of the companion architecture note: the Skyrme term is not fundamental; it is *generated* when the tower above the pion is integrated out, and the tower's ground floor — the ρ meson at 775 MeV, the first excited mode of the same flavor quantum numbers — dominates the sum. The prediction: $e \approx g_{\rho\pi\pi}$, the ρ's coupling, with the KSRF relation $m_\rho^2 = 2g^2 f_\pi^2$ tying the coupling to the masses. This note verifies both against data:

| Determination | Value | Source |
|---|---|---|
| $g_{\rho\pi\pi}$ from the measured ρ width | **5.976** | $\Gamma = g^2p^3/6\pi m_\rho^2$ with $\Gamma$ = 149.1 MeV, p = 361.6 MeV |
| g from KSRF, $m_\rho/(\sqrt{2}\cdot f_\pi)$ | **5.952** | $m_\rho$ = 775.26 MeV, $f_\pi$ = 92.1 MeV |
| e fitted (ANW 1983, massless pion) | **5.45** | nucleon-sector fit |
| e fitted (massive-pion fits) | ≈ 4.8 | later literature |
| ratio $e_{ANW}$ / g | **0.91** | — agreement at 9 percent |
| stiffness length unit $1/(eF_\pi)$ | 0.281 fm | vs $\hbar c/m_\rho$ = 0.255 fm (ratio 1.10) |

Two independent roads — the decay width and the mass relation — give the same g ≈ 6.0; the fitted stiffness sits 9 percent below it, inside the Skyrme model's honest 10–20 percent band; and the stiffness *length* is, to 10 percent, the ρ's own Compton cell. The sentence this buys, and it is the sentence of the note:

**The stiffness of a rung is the door to its next mode.** The Rattenschwanz that the seal amputates from the facade does not vanish without trace — it returns as a tower, and the tower's ground floor sets the stiffness coefficient of the storey below it. The cellar greets the facade with the address of its first floor.

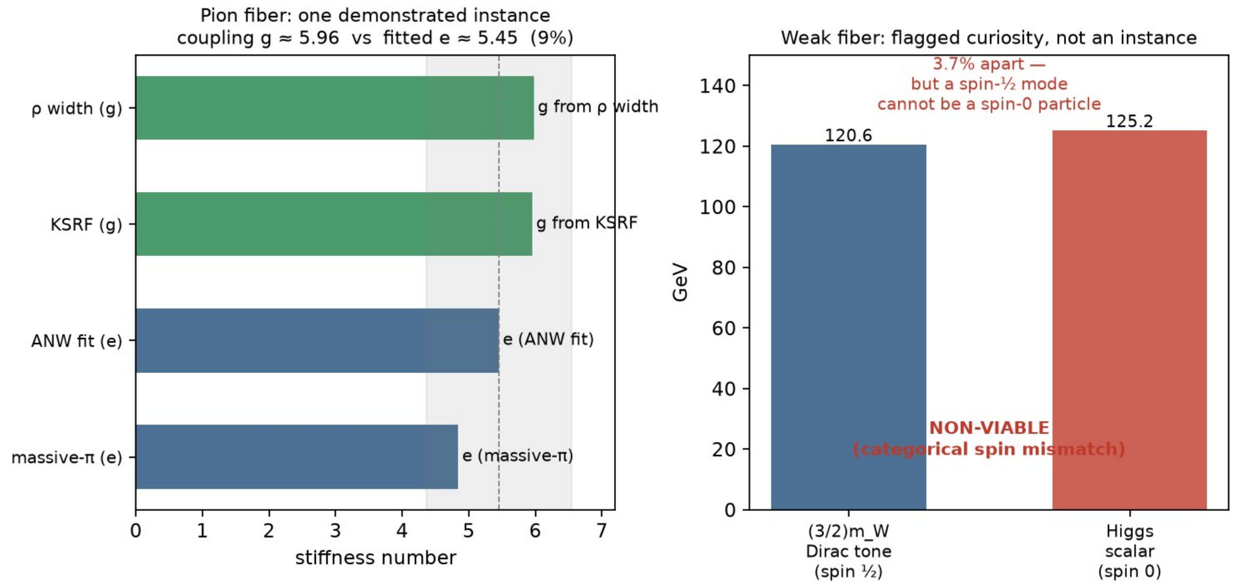


*Figure 1. Left: the pion fiber's stiffness number, four ways — two measurements of the first excited mode's coupling (green) against the fitted Skyrme coefficient (blue), agreeing inside the model's honest ±20% band. This is the one demonstrated instance. Right: the weak-fiber comparison — the lowest computed tone against the measured Higgs, 3.7 percent apart — shown only to be flagged NON-VIABLE: the tone is a Dirac (spin-½) eigenmode, the Higgs a scalar (spin-0); a categorical mismatch, not a bookkeeping detail.*

## 6. The Pattern, Translated Up the Ladder

If the pattern holds — stiffness coefficient of fiber n = coupling to its first excited mode — it should hold on every rung. Translate it to the weak fiber. Its harmonic tower was computed in the discrete-orbits note: the lowest Dirac tone of the wound $S^3$ of radius ħ/m_W c sits at

$$(3/2)\cdot m\_W = 120.6 \text{ GeV},$$

and the measured Higgs sits at 125.25 GeV — 3.7 percent away, the very curiosity that note filed under 'derive it or bury it.' It is tempting to read this as **the Higgs as the ρ of the weak winding**, and there is a fit of function to admire: stiffening the W's winding to its 2.5-attometre range *is* the Higgs's job in the standard model. But this paper must state plainly that the comparison is **not yet viable**, and not merely 'unresolved'. The computed weak-fiber tone is a **Dirac (spinorial) eigenmode**; the Higgs is a **scalar**. A spin-½ mode cannot *be* a spin-0 particle — this is a categorical mismatch, not a bookkeeping factor to be sorted out later. Until a genuinely scalar mode of the wound weak $S^3$ is exhibited near that energy, the 3.7-percent coincidence is a number without a mechanism, and it does **not** constitute a second instance of the pattern.

So, honestly: the stiffness-as-first-mode pattern has **one** demonstrated instance — the pion fiber, at 9 percent, with the ρ a genuine scalar-isovector state of the right quantum numbers. The weak 'instance' is recorded only as a flagged numerical curiosity, blocked by the spin mismatch above, and Eddington patrols every corridor where pure numbers meet measured masses.

## 7. Falsifiers and the Program

The pattern is exposed on three sides, and says so. (i) *QCD side:* if improved Skyrme fits (pion mass, quantum corrections) converge on a stiffness incompatible with g_ρ beyond the model's stated band, the anchor instance fails and the translation loses its ground. (ii) *Weak side:* the pattern requires the spin bookkeeping of the (3/2)m_W tone to be resolvable in the fiber's favor; a demonstration that no scalar mode of the wound $S^3$ sits near that energy kills the second instance. (iii) *Lepton side — the open case:* the electron's Q-ball needs its own tower; the pattern predicts its stiffness is set by the first excited lepton mode — and the muon, at 206.8 electron masses, is the obvious candidate tenant, though no calculation of a lepton-fiber tower yet exists in this series. Constructing it is the named next step, and it would convert the mass-spectrum null result of the condensation arc — masses off the α-rungs — into the pattern's third test.

The program, then, in one line each: derive the √2 of KSRF inside the frame formalism; resolve the weak tone's spin; build the lepton tower. The stiffness problem is no longer 'why is there a term' (Section 3 closed that) nor 'what sets its scale' (Section 5 gave the address); it is 'why does the next mode live where it lives' — which is the same question one rung higher. A ladder should feel exactly like this.

## 8. What Is Claimed, and What Is Not

**Claimed.** (i) The identity: the Skyrme term is the squared pullback of the field's *target* sphere $S^3$ curvature two-form — a standard differential-geometry result, verified numerically — so within the effective field the stiffness is the target's second geometric invariant, not an alien addition. This is expressly **not** a claim that the stiffness is the foundation theory's spacetime frame curvature $R = d\omega+\omega\wedge\omega$; those are two different manifolds' curvatures, and their identification is not made. (ii) The verified address: for the pion fiber, the coefficient equals the first excited mode's coupling at the 9 percent level, by two independent measurements. (iii) That the same 'first-mode' pattern is one *demonstrated* instance (pion/ρ), with the weak-fiber comparison recorded only as a flagged, non-viable numerical curiosity (Dirac tone vs scalar Higgs).

**Not claimed.** Priority: the geometric reading of the Skyrme term is established differential geometry; the ρ-generation of the term and the KSRF relation are literature (Skyrme 1961; Kawarabayashi–Suzuki–Riazuddin–Fayyazuddin 1966; Bando–Kugo–Yamawaki 1985; Adkins–Nauenberg–Witten 1983). The contribution here is the identification with the frame pair, the ladder translation, and the relocation of the problem. Also not claimed: any derivation of g itself, of KSRF's √2, or of the Higgs identification — the last remains a curiosity in quarantine, awaiting the spin bookkeeping it needs to live or the null result it needs to die.

## 9. The Sentence

The stiffness is the curvature of the winding's *target* sphere, squared — a clean geometric identity within the effective field, though not (this paper is careful to say) the foundation's spacetime curvature; and its coefficient is the door-price of the first mode above it — measured in the pion fiber at nine percent, with the ρ a genuine scalar-isovector state — so that at least the strong rung's rented pillar is revealed to be the next rung of the same ladder, while the weak-fiber echo remains a flagged coincidence, blocked by a spin mismatch, waiting for a scalar mode to earn or lose it.

## References

T. H. R. Skyrme, Proc. Roy. Soc. A 260, 127 (1961); K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966) — the KSRF relation; M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. 164, 217 (1988) — hidden local symmetry and the ρ-generated Skyrme term; G. S. Adkins, C. R. Nauenberg and E. Witten, Nucl. Phys. B 228, 552 (1983); G. H. Derrick, J. Math. Phys. 5, 1252 (1964); Particle Data Group values of $m_\rho$, $\Gamma_\rho$, $m_\pi$, $f_\pi$, $m_W$, $m_H$; and the papers and notes of this series (the Frame Quaternion note — θ, R, and the template; the Postulates — P2, the anatomy of a world; From Space Quaternion to Particle — Derrick's wall and the fitted e; Why the Orbits Are Discrete — the weak fiber's towers and the $(3/2)m_W$ curiosity; The Three Storeys — the seal and the returning Rattenschwanz). Verification script: stiffness_check.py. (Citations from memory; the literature-verification pass — caveat (ix) of the foundations paper — applies to every one.)

*Acknowledgment: derivation walk-through, numerical verification, figure and drafting by machine (Claude, Anthropic), in conversation; the path — 'prüfen, ob der Skyrme-Term nicht postuliert werden muss, sondern als nächster Term der Frame-Quaternion-Entwicklung automatisch dasteht' — was chosen by the author, and the reading is the author's. The geometry honored here belongs to Skyrme, to Kawarabayashi, Suzuki, Riazuddin and Fayyazuddin, and to Bando, Kugo and Yamawaki.*