Tolman from Staticity
The Temperature History Is a Theorem of P3 — and What It Cannot Decide

Two Independent Derivations of T·V = constant, the Redshift–Temperature Identity, and the Honest Standing of the Series' Most-Cited Audit

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It Is All One — Foundations (a note for the Allgemeine Feldtheorie). July 2026 (working draft)

Abstract

The temperature history $T(z) = T_0(1+z)$ is listed in this series' ledger as its cleanest passed test, and it is read there as an audit of the cosmology — of Metric D in particular. This note re-derives it from first principles by two independent routes and reports that the reading must be corrected in one respect and can be strengthened in another. The correction: the relation follows from Postulate P3 alone. Any static spacetime whatever satisfies it, because the Tolman–Ehrenfest theorem $T \cdot V$ = constant holds for every static metric, with V the lapse $\sqrt{(-g_{tt})}$; and in a static spacetime the redshift is defined by that same lapse, $1+z = V_{obs}/V_{emit}$, so $T(z) = T_0(1+z)$ is an identity between two appearances of one function rather than a prediction fitted to data. It therefore carries no power to discriminate among static metrics — it is blind to the areal radius R(r), which is the entire remaining content of Metric D — and, worse for its advertised role, the expanding cosmologies predict the same law for an unrelated reason (adiabatic cooling, $T \propto 1/a$). The measurement at z = 2.4 is common ground; it refutes neither party and supports neither over the other. The strengthening: the first derivation given here is native to this series rather than imported. It runs on P1's own machinery — time carried as an imaginary value on the quaternion's real axis, $W = i\tau$, so that thermal equilibrium is periodicity around the W-circle — and needs exactly one physical premise beyond staticity: that the system is in one equilibrium, so that the W-circle closes with a single global period. Local proper time runs slow by the lapse, so the proper circumference at each point is V times the global one, and temperature, being the reciprocal circumference, is proportional to 1/V. On this reading the family law and the Tolman history are not two results but one theorem read at two places: at a horizon it gives $T \cdot R = \hbar c/2\pi k_B$, across a static field it gives $T \cdot V$ = constant. What the theorem does not give is the normalization: it fixes the shape of the history and leaves T_0 free, which is precisely where the anchor problem lives. Both routes are verified symbolically in the companion script. Every flag flown.

1. What Is Being Claimed, and What It Is Worth

The relation in question is one line:

$$T(z) = T_0 \cdot (1 + z)$$

It says the same bath, looked at further away, is hotter in proportion to its redshift. The series has audited it at $z = 2.4$ and it passes. The question this note asks is not whether it is true but what its truth buys — and the answer turns out to be both less and more than the ledger currently records.

Less, because the relation is a consequence of staticity alone and is therefore blind to everything that distinguishes one static cosmology from another. More, because the cleanest derivation of it is not an import from 1930 but a two-line consequence of this series' own first postulate, which makes the family law and the temperature history the same theorem wearing different clothes.

Two derivations are given. They are independent — one thermodynamic, one algebraic — and they agree.

2. Route One: the W-Circle (this series' own machinery)

Postulate P1 carries time on the quaternion's real axis as an imaginary value, $W = i\tau$. Section 4 of the Allgemeine Feldtheorie has already drawn the consequence: multiplying by i is a quarter-turn, so time behaves like an angle, and an angle can close. A system at temperature T repeats itself when carried once around a W-circle whose circumference, measured in that system's own proper time, is

$$\beta_{proper} = \hbar / (k_B \cdot T)$$

This is the Kubo–Martin–Schwinger condition, and in this series it is not borrowed but structural: temperature IS the reciprocal circumference of the W-circle.

Now add P3. A static spacetime has a global time symmetry — one timelike Killing vector, hence one Killing time coordinate t shared by every observer, however deep in the field they sit. Continue t to imaginary values and the equilibrium state is a state periodic in that imaginary Killing time.

Here is the one physical premise, and it is worth stating alone because everything turns on it: the system is in a single equilibrium, so the W-circle closes once, with one global period in Killing time. Call it β_∞. Two regions with different Killing-time periods would not be one equilibrium; they would be two systems, and the Euclidean section would not close as a single manifold. Equilibrium is exactly the statement that there is one period.

The rest is kinematics. Proper time and Killing time are not the same clock: at a point where the lapse is $V = \sqrt{(-g_{tt})}/c$, a Killing-time interval dt is a proper-time interval $V \cdot dt$. So the circle that is β_∞ long in Killing time is

$$\beta\_proper(x) = V(x) \cdot \beta\_\infty$$

long in the proper time of an observer sitting at x. Combine with the KMS circumference and the lapse cancels out of nothing — it survives:

$$\hbar \,/\, (k\_B \cdot T(x)) = V(x) \cdot \beta\_\infty \quad \Longrightarrow \quad T(x) \cdot V(x) = \hbar \,/\, (k\_B \cdot \beta\_\infty) = \text{constant}$$

That is the Tolman–Ehrenfest relation, derived from P1's W-circle and P3's global time symmetry, with one premise named. Deep in the well the lapse is small, the proper circumference is short, and the system is hot; far out the circle is long and the system is cold. Temperature is the size of the time-circle, read upside down — the same sentence the family law is built on, applied now to a static field instead of a horizon.

The unification is worth recording explicitly. Take the family law's setting: a world of curvature radius R, whose horizon accelerates its residents at $a = c^2/R$. The Unruh period is $2\pi c/a = 2\pi R/c$, and the same KMS step gives $T \cdot R = \hbar c/2\pi k\_B$. Take the static-field setting and the same step gives $T \cdot V$ = constant. One theorem, two readings. The series did not need two mechanisms and does not have two.

3. Route Two: Hydrostatic Equilibrium (independent, and it needs no quantum mechanics)

The second route uses no algebra of this series and no KMS condition — only the conservation of energy–momentum, which any metric theory supplies. Take a general static, spherically symmetric metric and leave both of its free functions unspecified:

$$ds^2 = -c^2 \cdot e\^(2\Phi(r)) \cdot dt^2 + dr^2 + R(r)^2 \cdot d\Omega^2$$

$\Phi(r)$ is the potential — the lapse is $V = e\^\Phi$ — and $R(r)$ is the areal radius, the number that says how big a sphere at coordinate r actually is. Fill this spacetime with a photon gas at local temperature $T(r)$, so that $\rho c^2 = aT^4$ and $p = aT^4/3$, and demand hydrostatic equilibrium, $\nabla\_\mu \, T\^\mu\nu = 0$. The radial component reads

$$dp/dr = -(\rho c^2 + p) \cdot d\Phi/dr$$

which is the relativistic Euler equation: pressure gradients hold matter up against the potential, and the inertia being held up includes the pressure itself. Substitute the radiation equation of state. Both sides carry the same factor $(4/3)aT^4$ and it cancels, leaving

$$dT/T = -d\Phi \quad \Longrightarrow \quad T \cdot e\^\Phi = T \cdot V = \text{constant}$$

The same relation, from thermodynamics rather than algebra. The companion script performs this calculation symbolically for the general metric — it solves the conservation equation for $T(r)$ without being told the answer and returns $T(r) = C \cdot e\^(-\Phi(r))$ — and then verifies all four components of $\nabla\_\mu \, T\^\mu\nu$ vanish identically for Metric D with $T(r) = T_0 \cdot e\^(kr)$.

One feature of that calculation deserves to be lifted out of the appendix, because Section 5 turns on it: $R(r)$ never appears. It cancels from the conservation equation before any equation of state is chosen. The Tolman relation is a statement about the potential alone, and the areal radius is invisible to it.

4. Why Redshift and Temperature Carry the Same Factor

Now the step that decides what the audit is worth. In a static spacetime the gravitational redshift is not an independent piece of physics to be computed; it is the lapse, by definition. A photon of proper frequency ν_e emitted at r_e and received at r_o has

$$1 + z = \nu_e / \nu_o = V(r_o) / V(r_e)$$

because the Killing time between successive wave crests is conserved along the ray — that is what a time symmetry means — and each observer converts it to proper time with their own lapse. Put the observer at the origin, normalise $V(0) = 1$, and $1+z = 1/V(r_e)$.

Set that beside the result of Sections 2 and 3, $T(r)\cdot V(r) = T_0\cdot V(0) = T_0$:

$$T(r) = T_0 / V(r) = T_0 \cdot (1 + z)$$

The relation is exact, and it is exact for the same reason a tautology is: the redshift and the temperature ratio are the same function V, met twice. Nothing was fitted, nothing was measured, no property of the source entered, and no feature of the geometry beyond staticity was used.

This is the honest content of the audit. $T(z) = T_0(1+z)$ is not evidence that the cosmology is right. It is evidence that the cosmology is static and that the bath is in equilibrium — which is P3, which is a postulate, tested.

5. What the Test Cannot Decide — Stated Plainly

Three consequences follow, and the first two subtract from the ledger.

It cannot distinguish Metric D from any other static metric. Section 3 showed that $R(r)$ cancels. Metric D's whole remaining content, once the potential is fixed, is its areal radius $R(r) = r\cdot e^{-kr}$ — the function that produces the angular-diameter turnaround at $z = e-1$ and everything geometric that follows. Tolman is blind to it. Two static cosmologies with the same potential and wildly different $R(r)$ return exactly the same temperature history. So this audit constrains one of Metric D's two functions and says nothing whatever about the other.

It cannot distinguish this cosmology from an expanding one. In a Friedmann universe a blackbody bath cools adiabatically as $T \propto 1/a$, and since $1+z = 1/a$ there also, the prediction is $T(z) = T_0(1+z)$ — the same line, for an entirely unrelated reason. The measurements at $z \approx 2–3$ are usually written as $T = T_0(1+z)^{(1-\beta)}$ and quoted as bounds on β; both frameworks predict $\beta = 0$. A measured $\beta \neq 0$ would be trouble for both. This series and the standard account are, on this one line, in complete agreement, and the ledger should say so rather than claim the line as a victory.

What it does test is real, and it should be kept: that the deep bath is in thermal equilibrium in a static field, with no entropy production and no departure from the (1+z) law across the observed range. That is a genuine constraint on P3, it is passed, and it is worth keeping in the column — relabelled.

Recommended correction to the Allgemeine Feldtheorie. In the theorem tree, $T(z) = T_0(1+z)$ currently appears under P3 as 'the measured temperature history, the series' cleanest passed test.' The derivation stands; the gloss should be replaced by: a theorem of P3 and equilibrium alone, independent of the metric's areal radius, and predicted equally by the expanding cosmologies — a passed consistency check with no discriminating power. In the six-column ledger it remains an audit; it should not be counted as support for Metric D.

6. What the Theorem Does Not Fix: the Normalization

One more limitation, and it points at a wound the series already knows by another name.

The constant in $T \cdot V$ = constant is not determined by the theorem. Route one makes the reason transparent: the global period β_∞ is a free parameter of the equilibrium. In one familiar situation it is not free — a spacetime with a Killing horizon fixes it, because the Euclidean section must be smooth where the horizon closes, and that single regularity condition is what produces the Hawking and Gibbons–Hawking temperatures. Metric D has no such closure available at finite proper distance: its lapse e^{-kr} sinks toward zero only as $r \to \infty$, and the structure at $r = c/H$ is the angular turnaround and the closing of the exit cone, not a Killing horizon with a smoothness condition to impose.

So the theorem fixes the history's shape and leaves its normalization open. Every question of the form why this temperature and not another — the 2.725 K itself, the anchor problem, the quarantined geometric mean — lives entirely in that undetermined constant, and no amount of work on the shape will touch it. The Family Law's Cosmic Rung note reached the same place from the other side: the CMB's value routes through the recombination temperature and hence through η, the photon-to-baryon ratio, the one number this series still owes. Route one now says why that is structural rather than accidental: shape is a theorem, normalization is an input.

7. Status of This Note

What is established: that $T \cdot V$ = constant follows from P3 plus equilibrium, by two independent routes, one of them native to P1; that $T(z) = T_0(1+z)$ is then an identity, because the redshift is the same lapse; that the relation is blind to $R(r)$; and that it is predicted equally by the expanding cosmologies.

What is not established, and is not claimed: nothing here supports Metric D. This note removes an item from the evidence for the cosmology and reclassifies it. That is a subtraction, made deliberately, because a ledger that counts a tautology as a victory will misprice everything next to it.

What is owed, and is now the next task: the areal radius R(r). It is the entire remaining content of Metric D, Tolman cannot see it, and it needs a principle of its own — either derived from P2's seal, if the form $ds^2 = dr^2 + e^{(-2kr)}(-c^2dt^2 + r^2d\Omega^2)$ can be shown to be the seal applied uniformly to everything transverse to the fall, or taken as an input with the angular turnaround at $z = e-1$ named honestly as the observation that fixes it. What it must not be is fixed by the turnaround and then defended by it.

On authorship, since this series' first open item is precisely an independent re-derivation by the author: this note was drafted and its algebra verified by machine. It is a walkthrough to be checked, not a discharge of that item. The item stands until M.S. has walked both routes himself — which, for Route One, is three lines and an afternoon.

8. The Sentence

The temperature history is not a measurement the cosmology passed but a shape the postulate could not have avoided: one equilibrium closes the W-circle once, the lapse makes it shorter for whoever stands deeper, and the same lapse that reddens the light warms its source — so $T(z) = T_0(1+z)$ says only that the stage is eternal and the bath is at peace, tells us nothing about how big the spheres out there are, is said word for word by the expanding cosmologies we are arguing with, and leaves untouched the one number that would make it a prediction: how warm.

References

R. C. Tolman, Phys. Rev. 35, 904 (1930); R. C. Tolman and P. Ehrenfest, Phys. Rev. 36, 1791 (1930); R. Kubo, J. Phys. Soc. Jpn. 12, 570 (1957); P. C. Martin and J. Schwinger, Phys. Rev. 115, 1342 (1959); W. G. Unruh, Phys. Rev. D 14, 870 (1976); G. W. Gibbons and S. W. Hawking, Phys. Rev. D 15, 2738 (1977); and the papers and notes of this series (the Allgemeine Feldtheorie — the theorem tree and the six columns, §4 for the W-circle and the family law; Redshift as Infall / Metric D; De Sitter or Metric D; The Family Law's Cosmic Rung — the anchor and η; Curved One Way). Verification script: Cosmology/tolman_check.py (the general-metric derivation solved symbolically; all four conservation components verified for Metric D). (Citations from memory; the literature-verification pass applies to every one.)

Acknowledgment: the decision to attack Tolman first, and to treat the areal radius as the separate question it is, is the author's. Drafting and symbolic verification by machine (Claude, Anthropic).