The Algebra Budget: Why Colour Cannot Fit in One Quaternion

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Abstract. We ask a bookkeeping question: how much of known physics fits inside a single complex quaternion? We count, item by item, in plain language. The answer: spacetime and gravity, electromagnetism, the weak force, and the electron — everything except colour. Colour requires eight independent controls, and a quaternion's purse holds exactly three. The minimal enlargement mathematics permits is the octonion, which is nothing more exotic than a *pair* of quaternions multiplied with one twist — and the twist is not bookkeeping: it is confinement itself. The conclusion, stated once and defended throughout: the Standard Model is expressible entirely at the quaternion level — complex, and paired.

1. The Question

Cohl Furey and, before her, Geoffrey Dixon and Günaydin and Gürsey, showed that the symmetries of the Standard Model of particle physics can be found inside the octonions — the eight-part numbers. This series makes a sharper claim: most of the Standard Model never needed the octonion at all. It lives one floor down, in the complex quaternions, where this series began (Paper 1). Only one tenant — colour, the charge of the strong force — is too large for that floor. This note counts the rooms, shows exactly why colour does not fit, and shows that the enlargement which houses it is still built entirely out of quaternions. Nothing in this note requires more than careful counting.

2. The Currency We Are Counting

A quaternion is a number with four parts:

$q = w + x\cdot\mathbf{i} + y\cdot\mathbf{j} + z\cdot\mathbf{k}$

where w, x, y and z are ordinary numbers, and **i**, **j**, **k** are three different square roots of minus one, multiplying by Hamilton's rules ($\mathbf{i}\cdot\mathbf{j} = \mathbf{k}$, and order matters: $\mathbf{j}\cdot\mathbf{i} = -\mathbf{k}$). A *complex* quaternion lets the four parts w, x, y, z themselves be complex numbers — each carrying the ordinary imaginary unit i of school algebra, which commutes with everything. So a complex quaternion has eight real numbers inside it, and two different kinds of "imaginary": the commuting i, which this series uses to make time behave like an angle ($W = i\cdot\tau$, the Postulates paper), and the non-commuting **i**, **j**, **k**, which are the three rotation axes of space.

The thing we will count is *symmetry dials*: the number of independent ways a physical law can turn a state without changing anything measurable. Each force's messenger particles correspond one-to-one with these dials. Electromagnetism has one messenger (the photon): one dial. The weak force has three (the two charged W bosons and the Z): three dials. The strong force has eight (the gluons): **eight dials**. These numbers are measured facts, decades old, not choices.

3. The Inventory: What One Complex Quaternion Carries

Spacetime and gravity. Multiplying by unit complex quaternions produces every rotation of space and — because a boost is a rotation by an imaginary angle, courtesy of the commuting i — every change of velocity as well. That is the entire Lorentz group, the symmetry of special relativity, and it is where this series started: spacetime itself is the complex quaternion (Paper 1). The frame field equations note carries gravity on the same algebra.

Electromagnetism. Nail down one axis — the shutter axis, the plane in which a particle's internal clock turns (companion note on Schrödinger). The turns that survive the nailing are the turns about that single axis: a circle. A circle is one dial. One dial, one photon. Electromagnetism's group, called U(1) in the trade, is the circle of rotations about a chosen quaternion axis.

The weak force. Do not nail anything: let all three axes turn freely. Three axes, three dials. Three dials, three weak bosons. The weak force's group, called SU(2), *is* the unit quaternions — Hamilton's own sphere — and its notorious handedness (the weak force acts only on left-handed particles) is the algebra's own orientation: **i**·**j** = +**k** but **j**·**i** = −**k**. The order of multiplication in a nineteenth-century number system shows up as parity violation in a 1957 laboratory.

The electron. Two complex-quaternion components, coupled to each other with strength m (the mass), reproduce the electron's wave equation — the form found by Cornelius Lanczos in 1929, a year after Dirac. The g-factor of 2 rides on the double cover (the Pauli paper of this series), and the whole hydrogen spectrum follows (the Quantum Leap paper).

Total so far: relativity, gravity, light, radioactivity, and matter — on eight real numbers and their multiplication table.

4. The Counting Theorem: What Does Not Fit

Now try to move colour in. Colour needs eight dials — eight gluons, measured at accelerators for fifty years. Count what the quaternion has to offer, generously: the three rotation axes give three dials (the weak force already lives there), and the commuting i adds one circle more, for a total of four. **Four is less than eight.** There is no clever arrangement, because the counting reflects a hard fact of algebra: complex quaternions are secretly the same thing as 2-by-2 complex matrices, and the colour group, called SU(3), needs 3-by-3 matrices — it shuffles *three* colours. You cannot seat three colours in a two-seat theatre. However the seats are rearranged, one colour stands in the aisle.

This is not a nuisance of notation. It is the reason the strong force is a different *kind* of thing from everything in Section 3, and the reason this series' Papers 4 through 6 had to open a second universe to house it.

5. Confinement Is the Witness

Here is the remarkable part. Suppose colour *did* fit inside one quaternion. Quaternion multiplication, whatever its other quirks, is associative: $(a\cdot b)\cdot c$ always equals $a\cdot(b\cdot c)$. But this series derived confinement — the fact that no quark and no fractional charge has ever been seen alone — precisely from *non*-associativity: the coupling that cages quarks is the associator, the difference between $(a\cdot b)\cdot c$ and $a\cdot(b\cdot c)$, which is exactly zero in any quaternion (companion note, Free in Pairs, Caged in Triples). A one-quaternion colour force would therefore be an *unconfined* colour force: free quarks in every detector, naked charges of one-third in Millikan's oil drops. Fifty years of null searches say otherwise. **Every failed search for a free quark is a measurement that colour outgrew the quaternion.** The size of the algebra is not a theorist's taste; it has an experimental signature, and the signature is a cage.

6. The Repair: A Pair with a Twist

The smallest number system larger than the quaternions that still permits division is the octonions — that is Hurwitz's theorem, and the ladder of number systems ends there. But the octonion is less exotic than its reputation. By the Cayley–Dickson construction it is exactly an *ordered pair of quaternions* — call them (q_1, q_2) — multiplied by one rule:

$$(q_1, q_2) \times (q_3, q_4) = (q_1\cdot q_3 - q_4^*\cdot q_2 , q_4\cdot q_1 + q_2\cdot q_3^*)$$

where the star means quaternion conjugation: flip the signs of the **i**, **j**, **k** parts. Read the rule slowly. The left slot of the answer starts with the ordinary product $q_1\cdot q_3$ — and then subtracts a *twisted* term, in which the second members of each pair reach across, one of them conjugated. That reaching-across is the entire novelty of the octonion. It is what makes the pair non-associative even though each member is associative; it is the gluon cross-coupling of Paper 4; it is the associator of the confinement argument; and it is where the eight colour dials come from — the pair's symmetry group (called G_2, with fourteen dials) holds an eight-dial subgroup exactly where one quaternion's four could not. Every octonion calculation in Papers 4, 5 and 6 of this series was in fact executed as quaternion-pair arithmetic by this rule — nothing else was ever computed.

7. The Claim, Settled

So the claim survives in a precise and, we think, beautiful form. **The Standard Model is expressible entirely at the quaternion level — complex, and paired.** One complex quaternion carries spacetime, gravity, electromagnetism, the weak force and the electron. A twisted pair of them carries colour, and the twist is not a formality: it is confinement, the thirds of the quark charges, and the freedom of quarks at close quarters, all in one algebraic gesture. Where Furey's programme takes the octonion whole, this series splits the same content along its natural seam — the outer universe on one quaternion, the inner universe on its paired twin, the family law binding what the algebra separates. There is no ninth dial hiding anywhere, and none is needed: the budget balances.

References

[1] C. Furey, "Standard model physics from an algebra?", Ph.D. thesis, University of Waterloo (2015); Phys. Lett. B 785, 84 (2018). [2] G. M. Dixon, *Division Algebras: Octonions, Quaternions, Complex Numbers and the Algebraic Design of Physics*, Kluwer (1994). [3] M. Günaydin and F. Gürsey, J. Math. Phys. 14, 1651 (1973). [4] W. R. Hamilton, Proc. Roy. Irish Acad. 2, 424 (1844). [5] A. Hurwitz, Nachr. Ges. Wiss. Göttingen, 309 (1898). [6] J. C. Baez, "The Octonions," Bull. AMS 39, 145 (2002). [7] C. Lanczos, Z. Phys. 57, 447 (1929). [8] This series: Papers 1–6 and companion notes (the Postulates; Schrödinger from the Geometry; Free in Pairs, Caged in Triples). *(Citations from memory; the series' literature-verification pass applies.)*