Schrödinger from the Geometry: the Shutter, the Projection, and the Balanced Frame

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Abstract. The Schrödinger equation — the workhorse of all quantum mechanics — is usually postulated. Here it is derived, in four slow steps, from this series' geometry: a state quaternion turning at its natural rate, viewed through the projection that the algebra itself supplies. Along the way the equation's two standing mysteries acquire names. The imaginary unit i that sits so strangely in $i\hbar\cdot\partial\psi/\partial t$ turns out to be a specific quaternion axis — the axis of the particle's internal clock, the *shutter*. And the potential energy V turns out to be the lapse — the same clock-rate field whose differences, read across the cosmos, are redshift (the Ledger note). One field, two books: read N across space with light and it is redshift; read it in place with matter and it is potential energy. As a bonus, the derivation reveals what the standard equation quietly discards — and the discarded half is spin. Every step is verified symbolically in the companion script; every symbol is introduced before use.

1. The Claim

Factor out of a quantum state the one motion it always has — its internal Compton rotation, 1.2×10^{20} turns per second for an electron — and what remains of the wave equation, exactly and without insertion, is Schrödinger's:

$$i\cdot\hbar\cdot\partial\psi/\partial t = -(\hbar^2/2m)\cdot\nabla^2\psi + V\cdot\psi$$

with the i identified, the V identified, and nothing postulated. Here ψ is the *envelope* — the slow story of where the particle is; ħ is Planck's constant divided by 2π; m the particle's mass; ∇^2 the sum of second position-derivatives (how sharply the envelope curves in space); V the potential energy; and t time. The derivation is four steps, each walked in full.

2. The Four Steps

Step one — the wave equation from the frame. The frame operator of this series' field-equation note, $D = -(i/c)\cdot\partial\_t + \mathbf{i}\cdot\partial\_x + \mathbf{j}\cdot\partial\_y + \mathbf{k}\cdot\partial\_z$, packages the four spacetime derivatives into one quaternion object (the commuting i on the time slot, Hamilton's **i**, **j**, **k** on the three space slots). Multiplied against its conjugate it produces the classical wave operator — the object that says "disturbances travel at speed c." A state quaternion Q(x,t) that also winds on its fiber at the Compton rate $\omega = m\cdot c^2/\hbar$ (the winding *is* the mass — the discrete-orbits note) obeys the wave equation with that winding as its only extra term. In the trade this combination is called the Klein–Gordon equation; here it is simply “waves plus winding,” with the mass never inserted by hand.

Step two — factor the shutter. Write the state as an envelope times its clock:

$Q(x,t) = \psi(x,t) \times \exp(-e_3 \cdot \omega \cdot t)$

The exponential is the *shutter*: uniform rotation about one quaternion axis, e_3, at the Compton rate — the tick of Postulate 3’s movie. The envelope ψ is everything else: the slow story riding on the fast turning. And here is the projection: ψ is taken to live in the flat plane spanned by 1 and e_3 — the plane of the shutter itself. **That restriction is complex quantum mechanics.** A plane spanned by 1 and one square root of minus one is, precisely, the complex numbers: the i of every quantum textbook is e_3 wearing a mask — the axis of the particle’s own clock.

Step three — the balanced frame. Substitute the factored state into the wave equation and expand (the companion script does this symbolically, term by term). The enormous rest-energy terms — the ω^2 pieces, the 511,000 electron-volts of sheer rotation — **cancel identically**. This is Postulate 3 speaking inside quantum mechanics: every frame of the movie is balanced; the huge constant turning drops out of the books; what survives is the equation of the *difference between frames*. Schrödinger’s equation is the ledger of the small change.

Step four — the slow envelope. For envelopes that change slowly compared with the shutter (the non-relativistic condition — satisfied by everything in chemistry), one small term may be dropped, and what remains is exactly $i\hbar \cdot \partial\psi/\partial t = -(\hbar^2/2m) \cdot \nabla^2\psi$, with $i \equiv e_3$. How small is the dropped term? For an electron in an atom, about seven parts per million — which is exactly the size of the known relativistic corrections (fine structure). The derivation even locates its own error bars where the measured corrections live.

3. The Potential Is the Lapse

On a curved stage the shutter ticks against *local* time. A particle held at position x runs its clock at the local rate $N(x)$ — the lapse, the same pure number the Ledger note used for redshift: $N = e^{-kr}$ × the product of the well factors $\sqrt{1 - r_s/\rho}$. Factoring out one *universal* shutter leaves behind the local difference, and that difference enters the equation as a potential:

$V(x) = m \cdot c^2 \cdot (N(x) - 1)$

which, for shallow wells, is exactly the Newtonian $m \cdot \Phi$ of every mechanics course. So the potential energy in Schrödinger’s equation is the local exchange rate of the particle’s clock against universal time — **the same field whose differences along a light ray are the cumulative redshift.** One function N: read it across space and it is the Ledger; read it in place and it is V. For the electromagnetic rung, the circle fiber contributes the same way — a rotation rate $q \cdot \varphi/\hbar$ on its own axis (q the charge, φ the Coulomb potential) — giving the $q\varphi \cdot \psi$ term and, with it, the hydrogen spectrum on which the chemistry rung stands.

4. What the Projection Hides

Step two *restricted* the envelope to the plane of 1 and e_3. A full quaternion envelope has a second complex component — and the pair transforms as a doublet under rotations: **the projection's discarded half is spin.** Keep it, and the same four steps deliver the Pauli equation (Schrödinger plus spin), with the magnetic $\sigma \cdot B$ term arising from the non-commutativity of Hamilton's units rather than from any added magnetic moment: the electron's $g = 2$ for free, as the Pauli-exclusion paper's double-cover argument requires. And the *unreduced* equation — no slow-envelope approximation at all — is the quaternionic wave equation of Lanczos (1929) and Conway, cited in the Quantum Leap paper. The hierarchy: Lanczos–Conway (the full quaternion wave), Pauli (slow, both components), Schrödinger (slow, one component). Each is the previous with something averaged away; none is postulated.

5. Caveats, Honestly

(i) The algebraic reduction from waves-plus-winding to Schrödinger by factoring the fast rotation is textbook physics; what this note adds is not the algebra but the *identifications* — i as the shutter axis, V as the lapse, the cancellation as the balanced frame, spin as the projection's remainder. (ii) Quaternionic quantum mechanics has a serious literature (Finkelstein, Jauch and Speiser; Adler's 1995 monograph) and one known hard problem: composite systems — the mathematics of *two* particles' joint state — resist fully quaternionic formulation. Our position is lighter than Adler's: the dynamics is quaternionic, but amplitudes are always read in the complex projection along the shutter, where the standard two-particle mathematics lives. Whether that position survives a careful audit against entanglement is owed, and named in the State Octonion paper's programme. (iii) The shutter multiplies from the right; left and right must be kept straight (they are, in the script). (iv) Citations from memory; the literature pass applies.

6. The Sentence

Schrödinger's equation is what the quaternion wave equation looks like through a shutter: project the state onto the axis of its own turning, cancel the balanced frame, keep the small change. The i is the axis, ħ is the gear ratio, V is the lapse — and spin is everything the projection left behind.

References

E. Schrödinger (1926); C. Lanczos, Z. Phys. 57, 447 (1929); A. W. Conway (1937); D. Finkelstein, J. M. Jauch and D. Speiser (1962); S. L. Adler, *Quaternionic Quantum Mechanics and Quantum Fields*, Oxford (1995); and the papers and notes of this series (the Postulates; the field-equation note; the Ledger of the Way; the Pauli paper; the State Octonion paper;

verification: schroedinger_verification.py). *(Citations from memory; the literature-verification pass applies.)*